Elements of Photogrammetry

PART 4

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What This Text Covers . . .

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PART 4

Oblique Photography

Principles of Oblique Photography

Uses of Obliques

1. An oblique, which is an aerial photograph taken with the optical axis of the camera lens intentionally tilted from the vertical, may be a high oblique or a low oblique. High obliques are tilted sufficiently to include the horizon. Since they give wide coverage, they are often used to obtain data for maps to be plotted at a small scale. Low obliques, on which the horizon does not appear, are used to compile topographic maps with a relatively high degree of accuracy. Two common methods of using high obliques are the trimetrogon system and the Canadian-grid system. In each method three cameras are used. Low obliques may be obtained either with multiple-lens cameras or with single-lens cameras. When a pair of low obliques are taken in sequence along a flight line so that both photographs cover essentially the same area, they are called convergent photographs.

Apparent Horizon and True Horizon

2. The apparent horizon of a high oblique is the visible line of demarkation between the terrain and the sky. If the terrain is perfectly level, the apparent horizon will appear as a slightly-curved line near the upper edge of the photograph. This line is a trace of the curved surface of the earth. If the terrain is mountainous, and the topography therefore irregular, the apparent horizon will appear as a broken line in which the slight curvature may or may not be discernible. The true horizon of a high oblique is the intersection of the horizontal

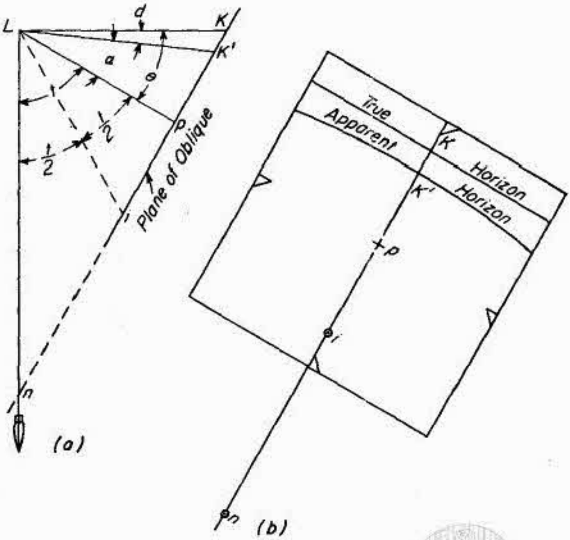

FIG. 1. HORIZONS AND DEPRESSION ANGLES

plane containing the exposure station and the plane of the photograph. The true horizon is an imaginary line. Therefore, its image does not appear on the negative.

In Fig. 1, view (*a*), the plane of the diagram is the principal plane of a high oblique taken at an exposure station *L*, and in view (*b*) the plane is that of the photograph itself. Since the true horizon is a plane, it appears as a straight line in each view. The point *K* represents the intersection of the true horizon and the principal line of the oblique. This principal line passes through the principal point *p* of the photograph and is perpendicular to the true horizon. The photographic nadir point is at *n*, and the isocenter is at *i*.

The apparent horizon in view (*b*) represents the surface of the earth where the terrain is perfectly level. It intersects the principal line of the photograph at *K′*. Because of atmospheric refraction, the line *LK′* in view (*a*) would actually be curved.

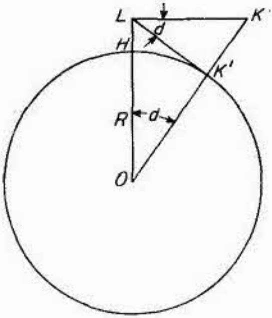

FIG. 2. DIP ANGLE

However, for the purpose of this discussion, the line LK' may be considered straight.

Dip Angle

3. The dip angle, designated as d in Figs. 1 and 2, is the angle measured in the principal plane of an oblique between the true horizon and the apparent horizon. The dip angle increases as the flying height becomes greater. In Fig. 2, L is an exposure station at a flying height H above the surface of the earth, O is the center of the earth, the line LK is the true horizon, and the line LK' is the apparent horizon. For simplicity, atmospheric refraction has been neglected. The sides of the angle formed at O between the radii to the exposure station L and to the point of tangency K' of the apparent horizon are perpendicular to the sides of the angle at L between the true and apparent horizons. Hence, the angle at O is also equal to d. If R denotes the radius of the earth,

$$\tan d = \frac{LK'}{OK'} = \frac{LK'}{R}$$

Since $LK' = \sqrt{\overline{LO}^2 - \overline{OK'}^2} = \sqrt{(R+H)^2 - R^2} = \sqrt{2RH + H^2}$,

$$\tan d = \frac{\sqrt{2RH + H^2}}{R}$$

The distance H is small compared to R, and this formula may be simplified as follows:

$$\tan d = \frac{\sqrt{2RH}}{R} = \sqrt{\frac{2}{R}}\sqrt{H} \tag{1}$$

Also, the dip angle is always small, and $\tan d = d'' \tan 1''$, in which d'' represents the dip angle, in seconds. Hence, formula 1 becomes

$$d'' = \frac{1}{\tan 1''}\sqrt{\frac{2}{R}}\sqrt{H} \tag{2}$$

Atmospheric refraction will decrease the value of d'' in formula 2. When the approximate average value of the radius of the earth, or 20.9×10^6 ft (feet), is used and the value of d'' is decreased to allow for refraction, formula 2 is reduced to

$$d'' = 58.8\sqrt{H} \tag{3}$$

in which d'' is the dip angle, in seconds, and H is the flying height above the datum, in feet.

The dip angle is given approximately by the relationship

$$d' = \sqrt{H} \tag{4}$$

in which d' is the dip angle, in minutes.

Depression Angle

4. The apparent depression angle of a high oblique is the angle measured in the principal plane between the apparent horizon and the optical axis of the camera lens. This is the angle α (alpha) in Fig. 1. Since pK' is perpendicular to Lp,

$$\tan \alpha = \frac{pK'}{f} \tag{1}$$

in which pK' is the distance measured on the photograph along the principal line between the principal point and the apparent horizon, in inches, and f is the principal distance of the camera, in inches.

Before the distance pK' can be measured, the principal line must be defined on the photograph. If two points on the apparent horizon, one at the left edge and one at the right edge of the photograph, can be identified, a chord can be drawn to join these two points, and the principal line can then be drawn through the principal point and perpendicular to the chord. If the ends of the apparent horizon cannot be defined easily, an average line is drawn through as many points on the apparent horizon as can be identified, and the principal line is then drawn perpendicular to this line.

The depression angle of a high oblique photograph is the angle measured in the principal plane between the true horizon and the camera axis. In Fig. 1 it is the angle θ (theta). Obviously,

$$\theta = \alpha + d \tag{2}$$

in which d is the dip angle.

Position of True Horizon

5. The true horizon may be drawn in its correct position on the photograph after the depression angle θ has been determined. As shown in Fig. 1,

$$pK = f \tan \theta$$

in which pK is the distance measured along the principal line between the principal point and the true horizon, in inches; and f and θ have the meanings given previously.

When the distance pK has been computed, it is laid off on the principal line to locate point K. The true horizon is then drawn through K in a direction perpendicular to the principal line.

Nadir Point and Isocenter

6. The nadir point, as n in Fig. 1, of an oblique is the point at which a plumb line passing through the exposure station pierces the plane of the photograph (extended). The nadir

point can be located on the downward side of the photograph by measuring from the principal point along the principal line a distance $f \tan t$, in which t is the tilt. Since $t = 90° - \theta$, the distance from the principal point to the nadir point is also equal to $f \cot \theta$. Then the distance from the true horizon to the nadir point is $f(\tan \theta + \cot \theta)$.

The isocenter, as i in Fig. 1, of an oblique is the point at which the bisector of the angle of tilt at the exposure station pierces the photograph. It can be located by measuring a distance $f \tan \frac{1}{2}t$ downward from the principal point along the principal line.

Also, the distance from the true horizon to the isocenter is $Ki = f(\tan \frac{1}{2}t + \tan \theta)$. It can be shown that the quantity in parentheses is equal to $\sec \theta$. Hence, the distance from the true horizon to the isocenter is $f \sec \theta$.

Scale of an Oblique Photograph

7. The scale of an oblique is constant along any line that is parallel to the true horizon, but varies along any other line. A line on an oblique passing through the isocenter and parallel to the true horizon would have the same scale as a vertical photograph taken with the same camera and from the same exposure station. If a series of constant-scale lines were drawn on an oblique, beginning at the true horizon and ending at the lower edge of the photograph, each succeeding line would have a greater scale than the preceding one. The scale of the oblique along one of these constant-scale lines is called the x-scale along the line. This scale is designated as s_x.

The scale of a line drawn perpendicular to the constant-scale lines – that is, parallel to the principal line – varies throughout the length of the line. The scale at a point on an oblique in a direction parallel to the principal line is called the y-scale at the point. This scale is designated as s_y.

In Fig. 3 a high oblique, for which the depression angle is θ, is taken at L from a flying height H above the datum. Point

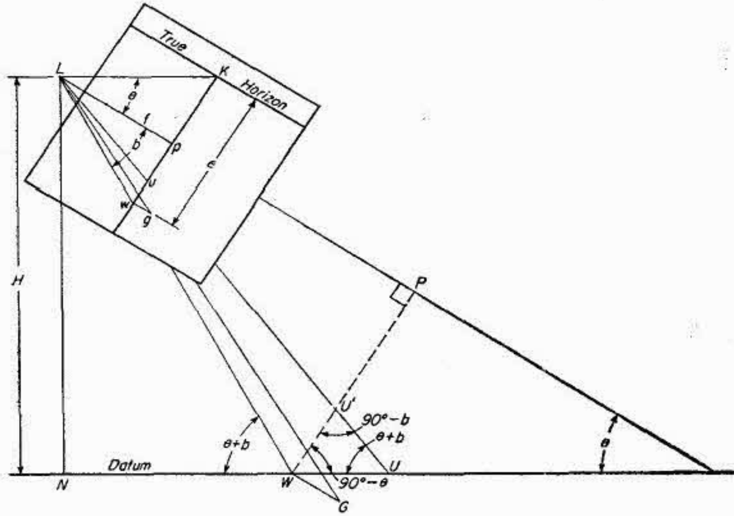

FIG. 3. SCALE OF OBLIQUE PHOTOGRAPH

w on the principal line is the image of the point *W* in the datum plane, and a ray to *w* makes an angle *b* with the optical axis *Lp*. Point *g* is the image of the point *G*, which is so located in the datum plane that the line *wg* is parallel to the true horizon on the oblique.

The *x*-scale along the line *wg* is

$$s_x = \frac{wg}{WG}$$

Since the triangles *wLg* and *WLG* are similar,

$$\frac{wg}{WG} = \frac{Lw}{LW} = \frac{(Lp)/\cos b}{(LN)/\cos\,[90° - (\theta + b)]}$$

Substituting *f* for *Lp* and *H* for *LN*, and simplifying, gives

$$s_x = \frac{f}{H}\,\frac{\sin(\theta + b)}{\cos b} \qquad (1)$$

in which s_x = scale of oblique along a line parallel to the true horizon, expressed as a ratio

f = principal distance of the camera, in feet

H = flying height, in feet
θ = depression angle
b = angle measured at the exposure station between the principal point and the point at which the scale line crosses the principal line

The angle b is considered positive if measured downward from the optical axis, and negative if measured upward from that axis.

The x-scale may be expressed in terms of the distance e, which is measured along the principal line from the true horizon to the line wg, and the depression angle θ. From Fig. 3,

$$e = f \tan \theta + f \tan b = f(\tan \theta + \tan b)$$

In applying this formula, the sign of b must be considered.

The preceding formula may be converted to

$$\frac{f \sin(\theta + b)}{\cos b} = e \cos \theta$$

Hence,

$$s_x = \frac{e \cos \theta}{H} \tag{2}$$

EXAMPLE. An oblique photograph is taken from a height of 8100 ft with a camera whose principal distance is 6.000 in. (inches). The distance measured on the photograph between the apparent horizon and the principal point is 2.730 in. Determine the scale along the apparent horizon and the scale along a constant-scale line through the principal point.

SOLUTION. The dip angle, in seconds, is

$$d'' = 58.8\sqrt{8100} = 5292''$$

So

$$d = 1^\circ\, 28'$$

Also, the apparent depression angle may be found from the relationship

$$\tan \alpha = \frac{2.730}{6.000} = 0.45500$$

Hence, $\alpha = 24^\circ\, 28'$, and the depression angle is

$$\theta = 24° \, 28' + 1° \, 28' = 25° \, 56'$$

The distance from the true horizon to the apparent horizon is

$$f \tan \theta - f \tan \alpha = 6(\tan 25° \, 56' - \tan 24° \, 28') = 0.188 \text{ in., or } 0.0157 \text{ ft}$$

Also, the distance from the true horizon to the principal point of the oblique is

$$f \tan \theta = 6 \tan 25° \, 56' = 2.918 \text{ in., or } 0.2432 \text{ ft}$$

The required scales are as follows:

At the apparent horizon, $s_x = \dfrac{0.0157 \cos 25° \, 56'}{8100} = 0.000001743$. Ans.

At the principal point, $s_x = \dfrac{0.2432 \cos 25° \, 56'}{8100} = 0.00002600$. Ans.

8. An expression for the y-scale at a point on an oblique may also be determined by referring to Fig. 3. Since the scale in the y-direction (parallel with the principal line) varies from point to point, the y-scale can be considered constant for only an extremely short distance. In Fig. 3 let the distance wu be a very small y-distance corresponding to the very small datum distance WU. The scale of this short segment is

$$s_y = \frac{wu}{WU}$$

If the line WP is made parallel to the principal line, the triangles LWU' and Lwu are similar, and

$$\frac{wu}{WU'} = \frac{Lw}{LW} = \frac{wg}{WG}$$

Since $\dfrac{wg}{WG} = s_x$,

$$\frac{wu}{WU'} = \frac{f \sin(\theta + b)}{H \cos b}$$

When WU is extremely small, the line LW may be assumed to