Elements of Photogrammetry

PART 3

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PART 3

Stereoscopy and Parallax

Geometry of Overlapping Vertical Photographs

Stereoscopic Vision

1. When a person with normal two-eyed vision looks simultaneously at two photographs which have been taken of the same scene from different viewpoints, using one eye for each photograph, he can visualize the scene in three dimensions. This phenomenon is called stereoscopic vision. In Fig. 1 the small circles at *a* and *b* represent the relative positions of the pupils of an observer's eyes, and the arcs *cd* and *ef* represent the corresponding retinas. If an object, such as *gh*, is at a considerable distance from the observer, the length *gh* will appear merely as a point *i* on each of the retinas. If the object *gh* is moved nearer to the observer, as at *g′h′*, the image on the retina *cd* will still be a point *i*, but the image on the retina *ef* will have a definite length *g″h″* and the point *h′* will appear to be nearer to the observer than point *g′*. The angles *A* and *B* are called the parallactic angles, or the angles of parallax; and the difference between angle *A* and angle *B*, or the angle *C*, is known as the differential parallax for the particular conditions.

Since the distance between a person's eyes is fixed, natural stereoscopic vision is possible only at distances ranging from about 10 in. (inches) to about 2000 ft (feet). The perception of depth can be obtained at a distance of less than 10 in. by the use of lenses. In order to be able to perceive depth at a distance greater than 2000 ft, a person must, in effect, increase the distance between his eyes. He may use a pair of binoculars or a range finder. Or a pair of photographs may be taken with

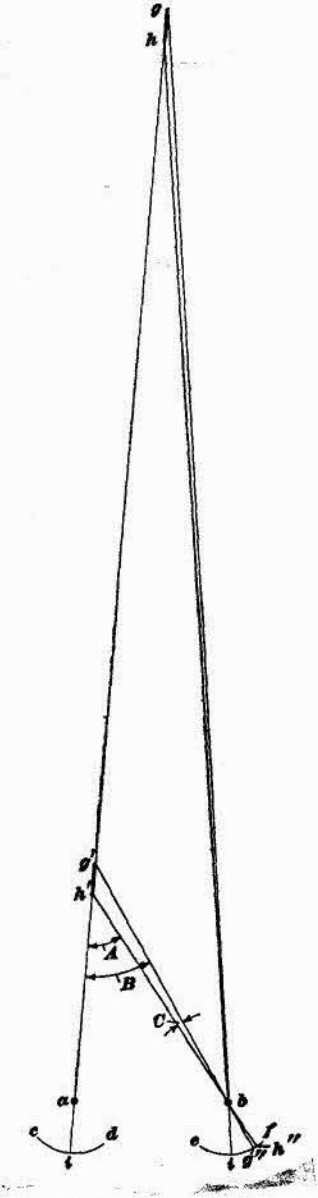

FIG. 1. STEREOSCOPIC VISION

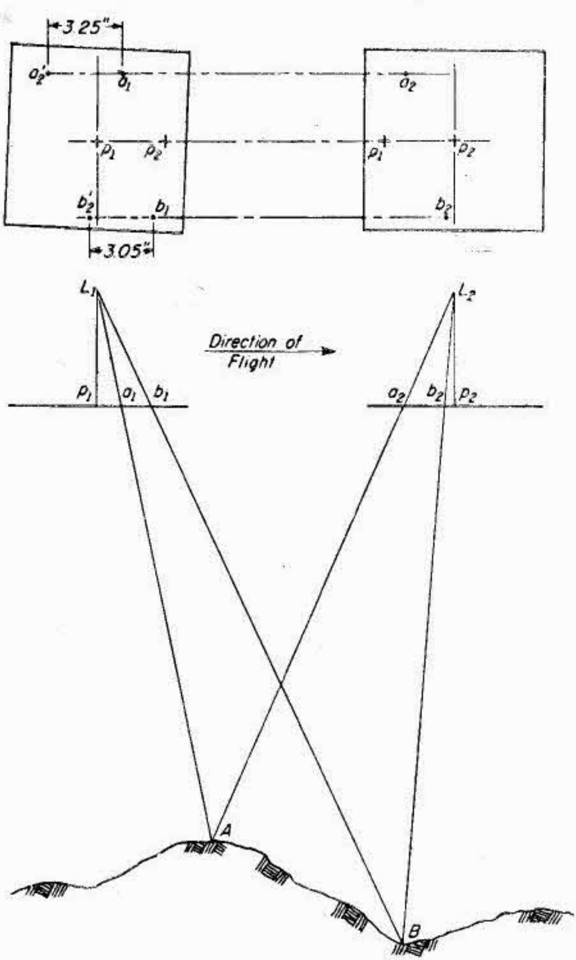

FIG. 2. PARALLACTIC DISPLACEMENT

cameras located at widely separated points, and these photographs may be viewed at the same time in a suitable manner. The simultaneous viewing of two such photographs is known as stereoscopy.

Parallactic Displacement

2. When a person looks at two stationary objects located at

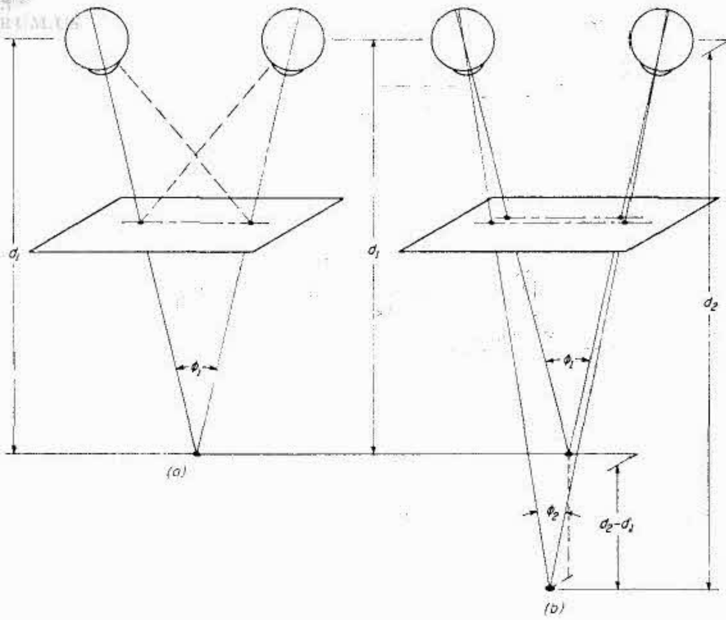

FIG. 3. STEREOSCOPIC VIEWING

different distances from his eyes and shifts his head sideways the two objects appear to move at different rates; in other words, one object seems to move with respect to the other object. The term *parallax* is used to refer to the apparent displacement of a stationary object with respect to other stationary objects when the observer changes his position.

If aerial photographs of the same area are taken from two different positions of the camera but from the same altitude the parallactic displacement of an object at a lower elevation will be less than that of an object at a higher elevation. In Fig. 2 it is assumed that the two photographs are obtained with the camera axis vertical and the camera at the same altitude. Hence, the scale at any point is the same for both photographs. The exposure stations are designated as L_1 and L_2; the princi pal points on the negatives are at p_1 and p_2; and the images of points *A* and *B* on the ground are at a_1 and b_1 on the left-hand photograph and at a_2 and b_2 on the right-hand photograph.

For the purpose of measuring parallax, it is necessary to locate the flight line p_1p_2 on each photograph and to assume as a *y*-axis a line that passes through the principal point of the photograph and is perpendicular to the flight line. These reference axes are not the same as those obtained by connecting opposite fiducial marks on the photograph.

When the *y*-axis of the right-hand photograph is made to coincide with the *y*-axis of the left-hand photograph and the principal points also coincide, the images a_2 and b_2 on the right-hand photograph occupy the positions a_2' and b_2' on the left-hand photograph. The distance a_1a_2' is the parallactic displacement of *A*, and the distance b_1b_2' is the parallactic displacement of *B*. It should be noted that the distance b_1b_2' is less than the distance a_1a_2'. Also, each of these distances is parallel to the line of flight through the principal points of the photographs.

Parallactic Angle

3. The principle underlying stereoscopic vision is illustrated in Fig. 3, (*a*) and (*b*). In view (*a*) it is assumed that two dots of similar size and shape have been drawn about 2 in. apart on a sheet of paper and that the sheet of paper has been placed on the top of a table or desk. When an observer looks downward at the sheet from a height of between 12 and 15 in., and concentrates his left eye on the left-hand dot and his right eye on the right-hand dot, he sees a single image of the two dots at a distance d_1 from his eyes. The impression of the distance d_1 is gained because of parallax. The angle ϕ_1 (phi) is the parallactic angle.

In view (*b*) a second pair of similar dots are added to the pair already on the paper. These new dots are located so that the distance between them is a little greater than the distance between the dots of the first pair and also so that the broken line joining them is parallel to the broken line joining the dots of the first pair. An observer viewing the second pair of dots stereoscopically gains the impression that these dots have a common image at a distance d_2 from his eyes. The parallactic angle ϕ_2 is shown to be smaller than the angle ϕ_1. If both pairs of dots are observed at the same time, the viewer apparently sees the two images at different distances from his eyes. Such stereoscopic viewing with the eyes alone is practical only under very favorable conditions.

Use of Stereoscope

4. When two overlapping aerial photographs are viewed stereoscopically, use is made of an instrument called a stereoscope, which greatly increases the angles of parallax. The two photographs are so arranged under the stereoscope that the left eye of the observer sees only a certain part of the left-hand photograph and the right eye sees only the part of the right-hand photograph that contains the corresponding terrain or objects. The visual effect is the same as though the observer were at a distance above the ground equal to the altitude of the aircraft during the flight and had his left eye at the point of exposure for the left-hand photograph and at the same time had his right eye at the point of exposure for the right-hand photograph.

If two adjacent aerial photographs are properly placed in a stereoscope, the actual shape and topography of the ground become apparent. The hills and hollows stand out in sharp relief, and the trees and buildings appear to be pointing upward and to have an exaggerated height. Also, images that appear to be hazy on individual photographs often become clear and distinct when viewed stereoscopically. The combined images of the two photographs appear as a relief map or model, generally called a stereoscopic model. Obviously

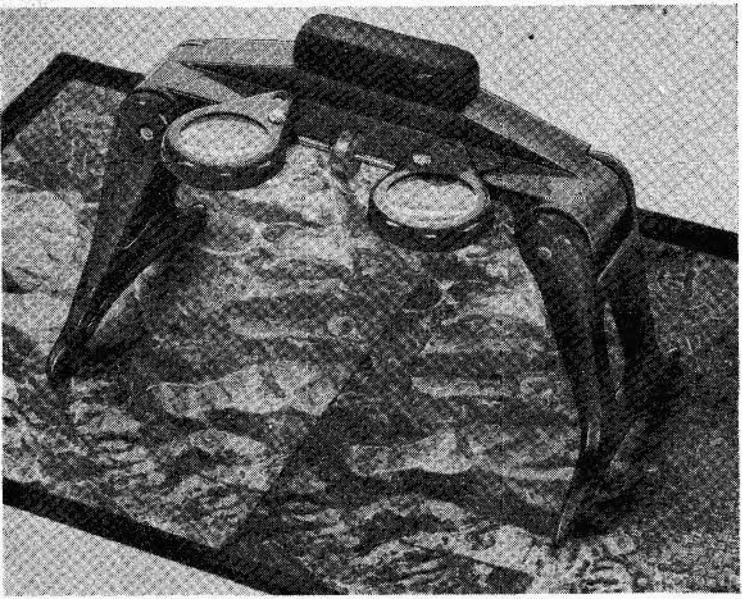

FIG. 4. LENS STEREOSCOPE

stereoscopic vision tends to facilitate the interpretation of aerial photographs.

Lens Stereoscopes

5. There are two basic types of stereoscopes for stereoscopic viewing of photographs, namely, the lens stereoscope and the mirror stereoscope. Each type has advantages and disadvantages. The lens stereoscope, shown in Fig. 4, consists of two simple magnifying lenses mounted with a separation equal to the average distance between the human eyes, but provision is made for changing this separation to suit the individual user. The lenses are mounted in a frame so that they are supported at a fixed distance above the table top. The distance between the nodal point of each lens and the plane of the table top, or photograph, depends on the focal length of the lenses. If the distance above the table is equal to the focal length of

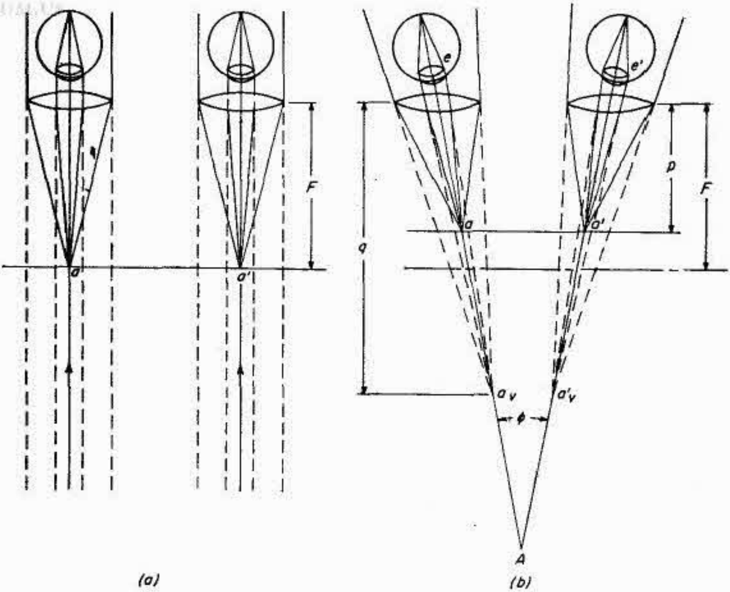

FIG. 5. PRINCIPLE OF LENS STEREOSCOPE

the lenses, the images of points on the photographs will appear to come from infinity. The separation of the two images of a point on the photographs will then be about equal to the distance between the eyes of the individual viewer for the most comfortable viewing. This condition is shown in Fig. 5, view (*a*). Rays of light from the images *a* and *a′* of a point, lying in the plane of focus of the lenses, appear to the eye to come from infinity. Thus, the optical axes of the eyes remain parallel, except for convergence due to differences in parallax between the different pairs of images.

If the photographs are placed inside the focal plane of the lenses, the images will appear to come from a finite distance. In such a case production of a parallactic angle compatible with the distance would require the separation of pairs of images on the photographs to be slightly less than the distance