USE OF TRIGONOMETRIC TABLE

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TRIGONOMETRIC CALCULATIONS

FUNCTIONS OF ANGLES

INTRODUCTION

1. Laying Off an Angle.—There are many times when the man in the shop is called on to lay off or to measure angles. One way of doing such work is to use a protractor. But, as the ordinary protractor is not graduated into divisions smaller than half a degree, or 30 minutes, it cannot be used to measure or to lay off accurately an angle containing a fraction of a degree that is more or less than 30 minutes. For example, suppose that an angle of 26° 34′ is to be laid off. An ordinary protractor graduated to half-degrees could be used to lay off 26° 30′, or $26\frac{1}{2}$°, but it cannot measure 26° 34′ with accuracy. Therefore, some other method must be employed whereby angles containing any number of minutes may be measured or laid out accurately.

2. One method of laying off an angle of 26° 34′ with accuracy is shown in Fig. 1 (*a*). A straight line *A B* is drawn to represent one side of the angle. This line may be of any convenient length, but in this case it is made 2 inches long. Let the side *A B* be indicated by *c*. At the end *B* of the line a perpendicular is drawn, and the distance *B C*, which is indicated by *a*, is made 1 inch; that is, $a = \frac{1}{2}c$. The points *A* and *C* are then joined by a straight line *A C*, indicated by *b*, and the

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angle *A* between the two lines *b* and *c* is then 26° 34′. In other words the angle 26° 34′ is accurately laid out by drawing two lines at right angles to each other, making one line half as long as the other, and drawing a third line to join the ends of the first two. It will be seen that *A B C* is a right-angled triangle with a right angle at *B*. As $a=1$ inch and $c=2$ inches, $a \div c = \frac{a}{c} = \frac{1}{2} = .5$; also, $a = \frac{1}{2}c = .5 \times c$. In this illustration, as well as in many of those that follow, the angles of a triangle will be denoted by the capital letters *A*, *B*, and *C*, and the sides opposite these angles by the corresponding small letters *a*, *b*, and *c*.

3. From what has been shown in the preceding article, it may be stated that if any right triangle is drawn, having one side

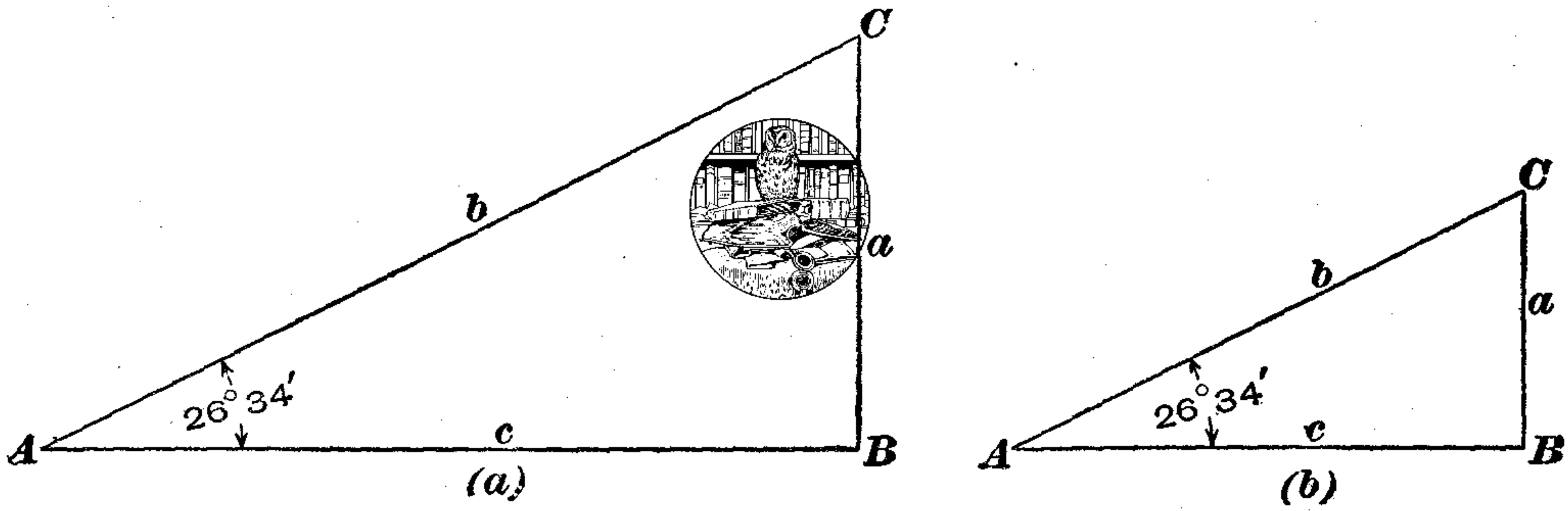

FIG. 1

.5 times the length of the other side, or half as long, the angle opposite the shorter side will be 26° 34′. This is true, no matter what length is chosen for the first side drawn. Suppose, for example, that the side *c*, or *A B*, in Fig. 1 (*b*) is made $1\frac{1}{4}$ inches long, or 1.25 inches, and that the side *a*, or *B C*, is drawn at right angles to *A B* and is made $.5 \times 1.25 = .625$ inch, or $\frac{5}{8}$ inch, long. Then, when *A C* is drawn to complete the triangle, the angle *A* between *b* and *c* will be 26° 34′, as before. If the side *c* were made 30 inches and the side *a* 15 inches, the result would be the same; that is, the angle *A* would be 26° 34′. Here, then, is a method that can be used in laying out accurately an angle of 26° 34′.

4. Tangent of an Angle.—It should be observed that the quotient obtained by dividing the length of the side *a* by the length of the side c is the same in the case of both triangles shown in Fig. 1; that is, $\frac{a}{c}=.5$, because $\frac{1}{2}=.5$ and $\frac{.625}{1.25}=.5$. The side a is the *side opposite* the angle A of 26° 34′ and the side c is the *side adjacent,* or next, to the angle A. Therefore, considering the angle A,

$$\frac{a}{c}=\frac{\textit{side opposite}}{\textit{side adjacent}}=.5$$

The ratio thus expressed is given the specific name of **tangent;** that is, the tangent of an angle is the ratio between the side opposite the angle and the side adjacent to the angle. For any one angle this tangent has a certain value that does not change. Thus, for an angle of 26° 34′, the value of the tangent is .5, because that is the value of $\frac{a}{c}$ or the value of the ratio of the side opposite to the side adjacent. This would be expressed in the following manner: tangent of 26° 34′ = .5.

5. Every angle has a tangent, the value of which is always the *same* for the *same angle,* but which changes when the size of the angle is changed. Thus, an angle of 6° 18′ has a tangent whose value is .1104; that is, if a right triangle is formed with one acute angle equal to 6° 18′, the ratio between the side opposite this angle and the side adjacent to it will equal .1104. Similarly, the tangent of 73° 25′ is 3.358; that is, if a right triangle is formed with one acute angle equal to 73° 25′, the side opposite the angle will be 3.358 times as long as the side adjacent to the angle, or, in other words, the ratio between the side opposite and the side adjacent will be 3.358. In any of the cases mentioned, the size of the triangle makes no difference. The ratio, or the value of the tangent, will be the same for a given angle, whether the triangle is large or small.

The values of the tangents of different angles have been found accurately to five decimal places and have been arranged in the form of a table, given at the end of this Section, so that, if the size of an angle is known, its tangent can quickly be found by referring to the table.

EXAMPLE.—Find the tangent of an angle if the side adjacent is 5 inches and the side opposite is $1\frac{7}{8}$ inches.

SOLUTION.—The tangent is the ratio of the side opposite to the side adjacent; hence, it is

$$1\tfrac{7}{8} \div 5 = 1.875 \div 5 = .375. \quad \text{Ans.}$$

6. Cotangent of an Angle.—A ratio can be formed between the side adjacent to an angle and the side opposite. Such a ratio is called the **cotangent** of the angle. In Fig. 1 (*a*), for example, the ratio of the side adjacent to the side opposite the angle A is the ratio of c to a, or 2 : 1, the value of which is $2 \div 1 = 2$; therefore, the cotangent of the angle A, or the cotangent of 26° 34′, is 2. But, the ratio of c to a, or the cotangent, is the reciprocal of the ratio of a to c, or the tangent; in other words, the cotangent is the reciprocal of the tangent, or 1 divided by the tangent. In the same way, the tangent is the reciprocal of the cotangent. For every angle there is a corresponding value of the cotangent, also, and the values of the cotangents have likewise been found and put in the form of a table.

EXAMPLE.—Find the cotangent of an angle if the side adjacent is 3.45 inches and the side opposite is 1.625 inches.

SOLUTION.—The cotangent is the ratio of the side adjacent to the side opposite, or the ratio of 3.45 to 1.625; hence, it is

$$3.45 \div 1.625 = 2.12308. \quad \text{Ans.}$$

7. Sine of an Angle.—The two ratios known as the tangent and cotangent are not the only ones that can be used. The ratio of the side opposite the angle to the hypotenuse may also be used. This ratio is called the **sine** of the angle. In the right triangle $A B C$, Fig. 1 (*a*), the hypotenuse is b and the side opposite the angle A is a. The ratio of the length of the side a to the length of the hypotenuse b, or $\frac{a}{b}$, is the sine of the angle A. For every angle there is a corresponding value of the sine, and the values of the sines, also, have been calculated and arranged in the form of a table. For example, if b were made 5 inches long, and a were made 1.8 inches long, the angle A would be 21° 6′. For, if $b = 5$ inches and $a = 1.8$ inches, then the ratio of a to b, or the sine of the angle A, is

1.8 : 5, the value of which is $1.8 \div 5 = .36$, which is the sine of the angle of 21° 6′. Therefore, the angle *A* would be 21° 6′.

EXAMPLE.—In a certain right triangle, the hypotenuse measures 3 inches and the side opposite the angle measures 1.875 inches. Find the sine of the angle.

SOLUTION.—The sine is equal to the ratio of the side opposite to the hypotenuse; hence, it is

$$1.875 \div 3 = .625. \quad \text{Ans.}$$

8. Cosine of an Angle.—Still another ratio that is useful in measuring or laying off angles accurately is that called the **cosine,** which is the ratio between the side adjacent to the angle and the hypotenuse. In Fig. 1 (*a*), for example, the cosine of the angle *A* is equal to the ratio of *c* to *b*, or the length of *c* divided by the length of *b*, or $\frac{c}{b}$. For every angle there is a definite value of the cosine, so that, by arranging the values of the cosine in a table, the value for any angle can quickly be found; or, the size of the angle corresponding to a certain cosine can easily be determined from the table.

EXAMPLE.—If the hypotenuse of a right triangle is 5 inches long and the side adjacent to the angle is 2.4375 inches long, find the cosine of the angle.

SOLUTION.—The cosine is equal to the ratio of the side adjacent to the hypotenuse; hence, it is

$$2.4375 \div 5 = .4875. \quad \text{Ans.}$$

9. Definition of Function.—When any two quantities are so related that a change in the value of one produces a change in the value of the other, they may be said to be functions of each other. In other words, a **function** of any quantity is a second quantity, the value of which depends on the value of the first quantity. As an illustration, consider the circumference of a circle in relation to the diameter. The circumference is equal to 3.1416 times the diameter. If the diameter is doubled, the circumference is made twice as great; and if the diameter is made half as large, the circumference becomes half as large. Therefore, the circumference is a function of the diameter, because its value depends on the value of the diameter. The area of a square is equal to the square of one of the sides; therefore, the area is a function of the length of a side, because it depends on the length of a side.

10. Trigonometric Functions.—In preceding articles, four ratios were found by using the three sides of a right triangle. The sine, cosine, tangent, and cotangent of an angle are called **trigonometric functions** of an angle. Each is a function of the angle because the value of the ratio depends on the size of the angle. If the angle is altered in size, the value of the ratio is altered; therefore, according to the definition of a function, the ratio is a function of the angle. The four ratios mentioned by name are called trigonometric functions because they are used in making calculations by **trigonometry,** which is a branch of mathematics that deals with the solving of problems that involve triangles. The word *trigonometry* comes from two Greek words: *trigonon,* meaning *a triangle,* and *metron,* meaning *measure.*

11. The trigonometric functions in most frequent use are the sine, cosine, tangent, and cotangent. When writing the names of these functions, however, it is customary to use abbreviations, to save time and space. For example, *sine* is abbreviated **sin;** *cosine* is abbreviated **cos;** *tangent* is abbreviated **tan;** and *cotangent* is abbreviated **cot.** Any one of these functions of an angle is expressed by writing the abbreviation and following that by the angle. Thus, the sine of 18° is written sin 18°; the cosine of 41° 3′ is written cos 41° 3′; the tangent of 12° 38′ is written tan 12° 38′; the cotangent of 59° 24′ is written cot 59° 24′; and so on. It should be noted that the abbreviations are *not* followed by periods, nor do they begin with capital letters. Also, the word *of* is not used in connection with the abbreviation. Thus, although *sin* is the abbreviation for *sine,* it stands for the two words *sine of,* because, for example, sin 27° is read *sine of 27°.* Similarly, *cos, tan,* and *cot* take the place of *cosine of, tangent of,* and *cotangent of.* For example, cos 19° is read *cosine of 19°*; tan 82° 17′ is read *tangent of 82° 17′*; and cot 43° is read *cotangent of 43°.*

12. The ratios that express the most commonly used functions may be grouped as follows, the angle to which they refer being the angle *A*, Fig. 1 (*a*):

RATIO	FUNCTION	ABBREVIATION
$\frac{\text{Side opposite}}{\text{Hypotenuse}}=\frac{a}{b}$	sine of A	$\sin A$
$\frac{\text{Side adjacent}}{\text{Hypotenuse}}=\frac{c}{b}$	cosine of A	$\cos A$
$\frac{\text{Side opposite}}{\text{Side adjacent}}=\frac{a}{c}$	tangent of A	$\tan A$
$\frac{\text{Side adjacent}}{\text{Side opposite}}=\frac{c}{a}$	cotangent of A	$\cot A$

The point to be kept in mind is that the sine of an angle is the ratio between the side opposite that angle and the hypotenuse; the cosine is the ratio between the side adjacent to the angle and the hypotenuse; and so on for the other functions. Suppose, for example, that the angle to be considered is the angle *C*, Fig. 1 (*a*). In this case the side *c* is the side opposite the angle, *b* is the hypotenuse, and *a* is the side adjacent. Then, the ratios for the angle *C* are:

RATIO	FUNCTION	ABBREVIATION
$\frac{\text{Side opposite}}{\text{Hypotenuse}}=\frac{c}{b}$	sine of C	$\sin C$
$\frac{\text{Side adjacent}}{\text{Hypotenuse}}=\frac{a}{b}$	cosine of C	$\cos C$
$\frac{\text{Side opposite}}{\text{Side adjacent}}=\frac{c}{a}$	tangent of C	$\tan C$
$\frac{\text{Side adjacent}}{\text{Side opposite}}=\frac{a}{c}$	cotangent of C	$\cot C$

TABLE OF TRIGONOMETRIC FUNCTIONS

EXPLANATION OF TABLE

13. Natural Functions.—From what has already been stated with regard to the trigonomteric functions of angles, it is plain that for a given angle there is only one value for its sine; also, there is only one value for its cosine; and so on for each of the other functions. Thus, it is possible to arrange a table giving the values of all the angles that are likely to be used in practical work and showing the values of the sine, cosine, tangent, and cotangent of each angle. Such a table will be found at the end of this Section. It gives the values of functions, to five decimal places, for all angles from 0° to 90°. These functions are called the **natural functions** of the angles, and comprise the natural sines, natural cosines, natural tangents, and natural cotangents. They are called natural functions in order to distinguish them from other classes of functions used in other kinds of calculations. The table is useful in working out many kinds of problems, as will be explained later.

14. Arrangement of Trigonometric Table.—The table at the end of this Section is divided into two parts. The first nine pages of the table contain the values of the natural sines and cosines, and the remaining nine pages contain the values of the natural tangents and cotangents. The columns containing the values of sines and cosines are marked Sine and Cosine, respectively, and those giving the values of tangents and cotangents are marked Tang and Cotang, respectively. For all ordinary calculations it is sufficiently accurate to take the value of an angle to the nearest minute. The table is therefore arranged so as to show the values of the functions for all angles between 0° and 90°, increasing 1′ at a time. All angles expressed in whole numbers of degrees from 0° to 44° will be found in the rows at the tops of the pages, the minutes being found in the first column at the left-hand edge of the page, reading downwards. The values of the functions are given in the columns headed by the names of the functions. For angles from 45° to 89° it is necessary to look along the bottoms of the pages, the minutes being given in the right-hand column, reading *upwards*. As 90° is equal to 89° 60′, the values of the functions of 90° will be found on the first and tenth pages of the table, at the top of the second and third columns.

15. The table gives the values of the functions of angles expressed in degrees and minutes only; that is, seconds are not here taken into consideration, because sufficient accuracy can be obtained, in solving the examples that follow, without using seconds in dealing with angles. In many other classes of work, however, it is necessary to consider seconds, to insure the desired degree of accuracy. In this Section, therefore, if an angle is given in degrees, minutes, and seconds, it should be taken to the nearest minute. As there are 60 seconds in 1 minute, the half-way point is 30 seconds. So, if an angle contains less than 30″, ignore the number of seconds; but if it contains 30″ or more, add 1 to the number of minutes and drop the seconds. These points may be made clear by examples. For instance, consider the angle 12° 27′ 45″. As 45″ is more than 30″, and therefore more than half a minute, add 1 to the number of minutes, making it $27+1=28$, and drop the 45″. Then the angle will be taken as 12° 28′; that is, 12° 27′ 45″ is nearer to 12° 28′ than to 12° 27′. Similarly, 65° 48′ 22″ is taken as 65° 48′, because 22″ is less than 30″ and is ignored; 7° 18′ 32″ becomes 7° 19′, as 32″ is greater than 30″; 47° 52′ 8″ is taken as 47° 52′; 29° 6′ 51″ is taken as 29° 7′; 35′ 30″ is taken as 36′; and so on.

16. Limits of Natural Functions.—Between 0° and 90° the sine increases from zero to 1, while the cosine at the same time decreases from 1 to zero, as may be seen by referring to the table. The tangent of an angle of 0° is zero, from which value it increases as the angle increases until, for an angle of 90°, its value is *infinity*. This word simply means a value so great that it cannot be measured. The cotangent of 0° is infinity and its value decreases as the angle increases, until it becomes zero for an angle of 90°.

APPLICATION OF TABLE

17. Finding Functions of Angles.—The values of the various functions of angles are never calculated directly, except for the purpose of making a table of such values. In making calculations that involve the use of sines, cosines, etc., a table such as that given at the end of this Section is employed, and the values of the functions are taken directly from it. The method of finding the values of the functions of an angle may be stated in the form of rules, as follows:

Rule I.—I. *If the given angle is less than 45°, locate in the table the figures that represent the number of degrees in the angle, as found at the top of the page, and below them find the column headed by the name of the function to be found.*

II. *Look down the column at the left-hand edge of the page until the figures representing the number of minutes in the given angle are found.*

III. *In the column headed by the name of the function, and on the same horizontal line as the number of minutes in the angle, note the number given, which is the value of the function to be found.*

Rule II.—I. *If the given angle is 45° or more, locate in the table the figures that represent the number of degrees in the angle, as found at the bottom of the page, and above them find the column marked with the name of the function to be found.*

II. *Look up the column at the right-hand edge of the page until the figures representing the number of minutes in the given angle are found.*

III. *In the column marked with the name of the function (at the bottom) and on the same horizontal line as the number of minutes in the angle, note the number given, which is the value of the function to be found.*

The way in which the foregoing rules are applied will now be illustrated by the solution of a number of examples.

EXAMPLE 1.—Find the sine of an angle of 37° 24′.

SOLUTION.—The sines and cosines of angles are given in the first half of the table. As the angle 37° 24′ is less than 45°, look along the tops of the pages in the first half of the table until the value 37° is discovered. Under this value are two columns, one headed Sine and the other Cosine. As the sine of the angle is to be found, its value will be in the column headed Sine. When the number of degrees in the angle is given at the *top* of the page, as in this case, the minutes in the angle must be located in the first column at the left-hand edge of the page. Therefore, look down this column (which is headed ′) until the number 24 is discovered; this is the number of minutes in the given angle. Then follow horizontally along the row of figures to the column headed Sine, under the value 37° already located. Here the number .60738 appears, and this is the value to be found; that is, sin 37° 24′=.60738. Ans.

EXAMPLE 2.—What is the cosine of 18° 52′?

SOLUTION.—The values of the cosine are given in the first half of the table. As the angle 18° 52′ is less than 45°, look along the tops of the pages until 18° is discovered. As the cosine is to be found, its value will be in the column headed Cosine, under 18°. Run down the left-hand column until 52, the number of minutes in the angle, is found. Then proceed horizontally to the right, to the column headed Cosine, under 18°, where the number .94627 appears. This is the value to be found; that is, cos 18° 52′=.94627. Ans.

EXAMPLE 3.—What is the value of tan 41° 8′?

SOLUTION.—The angle whose tangent is to be found is less than 45°. Therefore, run along the tops of the pages in the *latter* half of the table, which gives tangents and cotangents, until the value 41° is located. Follow down the left-hand column to 8, the number of minutes in the angle, and then proceed to the right along a horizontal line to the column headed Tang, under the value 41° already located. Here will be found the number .87338, which is the tangent of 41° 8′. Ans.

EXAMPLE 4.—Find the value of cot 29° 40′.

SOLUTION.—The values of cotangents are given in the latter half of the table. As the angle is less than 45°, run along the tops of the pages in the latter half of the table until 29° is discovered. Then run down the left-hand column to 40, the number of minutes in the angle. From 40 proceed to the right to the column headed Cotang under 29°, where the number 1.75556 appears. Then, cot 29° 40′=1.75556. Ans.

EXAMPLE 5.—What is the sine of 77° 43′?

SOLUTION.—The values of sines are given in the first half of the table. But as the angle 77° 43′ is greater than 45°, it is necessary to look along the *bottoms* of the pages until the value 77° is discovered.

When the whole number of degrees is found at the bottom of the page, the number of minutes must be located in the column at the *right-hand* edge of the page. In this column, therefore, locate 43, the number of minutes in the angle, and then proceed horizontally to the *left* to the column marked Sine at the *bottom* and having 77° beneath it. Here the value .97711 appears; that is, sin 77° 43′=.97711. Ans.

EXAMPLE 6.—Find the value of cos 82° 3′.

SOLUTION.—The first half of the table must be used; but, as 82° 3′ is greater than 45°, it is necessary to follow along the bottoms of the pages until 82° is discovered. Run up along the right-hand column until 3, the number of minutes, is located, and then proceed horizontally to the left to the column marked Cosine at the bottom and having 82° under it. Here will be found .13831, which is the cosine of 82° 3′. Ans.

EXAMPLE 7.—Find the tangent of 50° 21′.

SOLUTION.—Tangents are found in the latter half of the table. As 50° 21′ is greater than 45°, look along the bottoms of the pages in the latter half of the table until 50° is discovered. In the right-hand column locate 21, the number of minutes, and then proceed to the left horizontally to the column marked Tang at the bottom and having 50° under it. Here the value 1.20665 is found; that is, tan 50° 21′=1.20665. Ans.

EXAMPLE 8.—Find the cotangent of 66° 6′.

SOLUTION.—The latter half of the table must be used to find the cotangent of an angle. As 66° 6′ is greater than 45°, follow along the bottoms of the pages until 66° is discovered. Next, run up along the right-hand column until 6 is found, and then proceed horizontally to the left to the column marked Cotang at the bottom and having 66° beneath it. Here the value .44314 is found. Therefore, cot 66° 6′=.44314. Ans.

EXAMPLE 9.—What is the sine of 30°?

SOLUTION.—The given angle contains 30° and no minutes; hence, it may be written 30° 0′. As this is less than 45°, look along the tops of the pages in the first half of the table until 30° is discovered. Next, locate 0 in the left-hand column. It will be found in the first line. Follow along this line to the right, to the column headed Sine under 30°, where the value .50000 will be found. Then, sin 30°=.50000. Ans.

EXAMPLE 10.—Find the sine of 17° 31′ 56″.

SOLUTION.—According to Art. **15**, this angle is taken as 17° 32′, because 56″ is greater than half a minute, or 30″. Then, on looking in the first half of the table, in the column headed Sine. under 17°, and in the line having 32 at its left-hand end, the value .30126 is found. Hence, sin 17° 31′ 56″=.30126, approximately. Ans.

EXAMPLE 11.—What is the cosine of 37′ 37″?

SOLUTION.—There are no degrees in this angle; so it may be written 0° 37′ 37″. As 37″ is more than 30″, or half a minute, the number of minutes in the angle is increased by 1, and the given angle then becomes 0° 38′, approximately. The cosine of 0° 38′ is found in the first half of the table, in the column headed Cosine, under 0°, and in the line having 38 at its left-hand end. The value there found is .99994, which is the cosine of 37′ 37″, approximately. Ans.

EXAMPLE 12.—What is the tangent of 59° 42′ 11″?

SOLUTION.—According to Art. **15**, the 11″ is ignored, because it is less than 30″. The angle then becomes 59° 42′. This is greater than 45°, and so the value 59° is located at the bottom of the page in the latter half of the table. In the column above it, marked Tang, and on the same line as 42 in the right-hand column, will be found the value 1.71129, which is therefore the approximate value of the tangent of 59° 42′ 11″. Ans.

18. Finding Angles From Given Functions.—In some cases the value of a function of an angle is known and it is necessary to find the angle corresponding to that value. The table of trigonometric functions may be used for this purpose, but the method of procedure is the reverse of that used in finding the functions of angles. The table may not, and probably will not, contain the exact value of the given function in every case; but it will be accurate enough here to locate in the table the value nearest the given value of the function and then find the angle that corresponds to it. The following rule should be observed:

Rule.—I. *If the given function is the sine or the cosine, use the first half of the table; but if it is the tangent or the cotangent, use the latter half of the table.*

II. *In the proper half of the table search until the given value is discovered, or the nearest value to it.*

III. *If the name of the given function is at the top of the column in which this value is found, note the number of degrees at the top of the column, and to it annex the number of minutes given in the left-hand column, on the same horizontal line as the value of the function.*

IV. *If the name of the function is at the bottom of the column in which this value is found, note the number of degrees* *directly beneath that column and to it annex the number of minutes given in the column at the right-hand edge of the page and on the same horizontal line as the value of the function.*

V. *The combined number of degrees and minutes thus found will be the angle required.*

The following examples will illustrate how the rule may be applied:

EXAMPLE 1.—Find the angle whose sine is .57691.

SOLUTION.—The values of sines are given in the first half of the table. Search through the columns headed Sine until the value .57691 is discovered. It will be found in the column bearing 35° at the top and in the line having 14 at the left-hand end. Therefore, the angle whose sine is .57691 is 35° 14′. Ans.

EXAMPLE 2.—What angle has a sine whose value is .79300?

SOLUTION.—The values of sines are given in the first half of the table. Search through the columns headed Sine. The greatest value discovered will be .70711, which is at the end of the first half of the table and is the sine of an angle of 45°. As the given value, or .79300, is greater than .70711, the angle must be greater than 45°, because the sine increases as the angle increases. Therefore, it is necessary to continue the search through the columns having Sine at the *bottom*. Following this plan, the value .79300 is discovered in the Sine column having 52° under it at the bottom of the page and on a line with 28 in the right-hand column. Then, the required angle, having a sine of .79300, is 52° 28′. Ans.

EXAMPLE 3.—The cosine of an angle is .36120. What is the angle?

SOLUTION.—Cosines are given in the first half of the table. Search through the columns headed Cosine, and the least value found will be .70711, which is still much greater than .36120. This indicates that the search must be continued through the columns having Cosine at the bottom. Following this plan, it will be discovered that the exact value .36120 does not appear in the table; but two values are found that are close to it. These are .36135 and .36108. The first of these is .00015 greater than .36120 (because .36135−.36120=.00015) and the second is .00012 less than .36120 (because .36120−.36108=.00012). Thus, the second value is closer to .36120 than is the first one, and so the angle corresponding to .36108 is taken. As .36108 is in the Cosine column having 68° below it, and on the horizontal line having 50 at the right-hand end, the angle is 68° 50′. Ans.

EXAMPLE 4.—Find the angle whose tangent is .77777.

SOLUTION.—Tangents are given in the latter half of the table. Search through the columns headed Tang fails to show the exact value .77777, but the values .77754 and .77801 are discovered. The first of these differs from .77777 by .00023 and the latter differs from .77777 by .00024. Therefore, the first is the closer and is the value to be used. It is found in the Tang column having 37° at the top and in the line having 52 at the left-hand end. Therefore, the required angle is 37° 52′.

EXAMPLE 5.—The cotangent of an angle is .41144. Find the angle.

SOLUTION.—Cotangents are given in the latter half of the table. Search through the columns headed Cotang fails to show any value less than 1.00000, which is much greater than .41144. Therefore, it is necessary to continue the search through the column having Cotang at the *bottom*. Even then the exact value does not appear; but .41149 is discovered as the nearest value to .41144. The value .41149 is in the Cotang column having 67° at the bottom and in line with 38 in the right-hand column. Therefore, the required angle is 67° 38′. Ans.

EXAMPLES FOR PRACTICE

1. Find the sine of 48° 17′. Ans. .74644
2. Find the cosine of 13° 11′. Ans. .97365
3. What is the sine of 72°? Ans. .95106
4. Find the value of cos 51° 9′. Ans. .62728
5. What is the tangent of 30°? Ans. .57735
6. Find the cotangent of 82° 50′. Ans. .12574
7. (*a*) Of what angle is .26489 the sine? (*b*) Of what angle is it the cosine? Ans. (*a*) 15° 22′; (*b*) 74° 38′
8. (*a*) Of what angle is .68814 the sine? (*b*) Of what is it the tangent? Ans. (*a*) 43° 29′; (*b*) 34° 32′
9. The cosine of an angle is .36257. What is the angle? Ans. 68° 45
10. The tangent of an angle is 1.43916. What is the angle? Ans. 55° 12′
11. Find the angle whose cotangent is 2.12200. Ans. 25° 14′
12. Find the sine of 74° 36′ 58″. Ans. .96417
13. What is the cosine of 53′ 42″? Ans. .99988
14. What is the sine of 28′ 12″? Ans. .00814
15. Find the cotangent of 65° 7′ 40″. Ans. .46348

PRACTICAL TRIGONOMETRIC CALCULATIONS

19. The use of trigonometric functions in making various calculations may be most easily shown by illustrative examples such as the following:

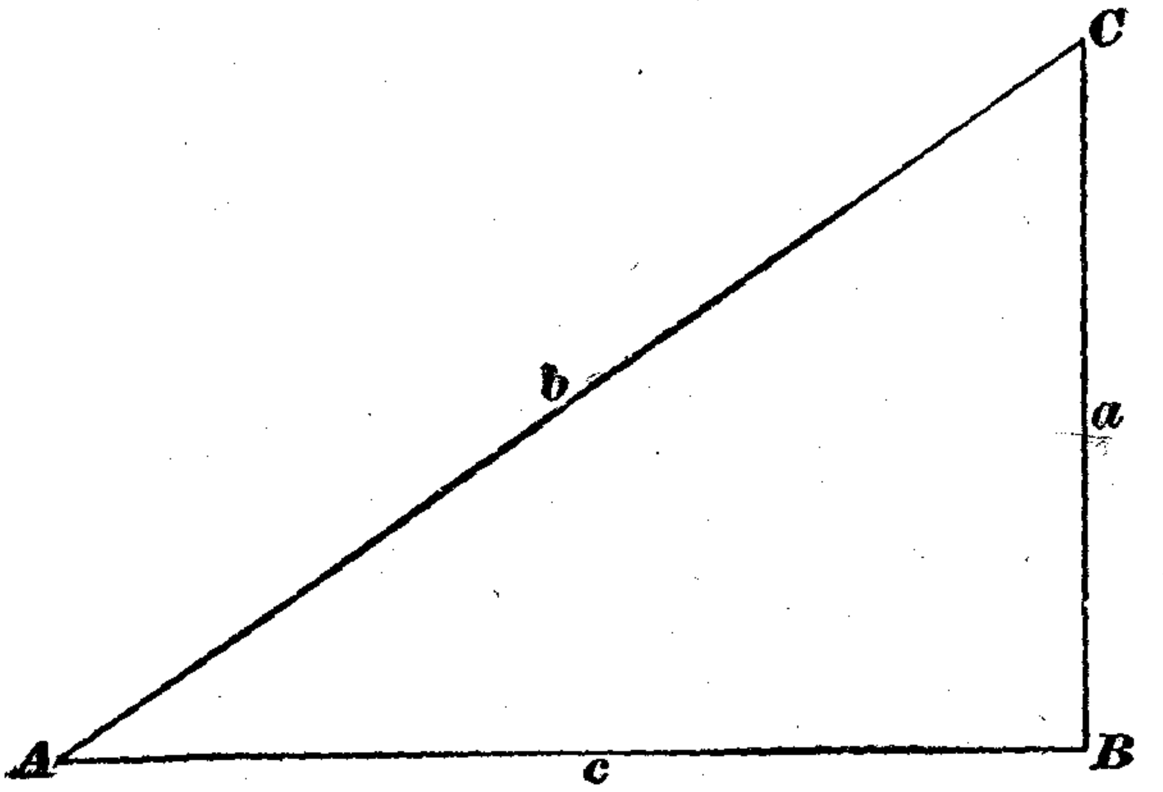

Fig. 2

Example 1.—Lay off two lines $A B$ and $A C$ so that the angle A between them will be 34° 31′.

Solution.—The two lines $A B$ and $A C$ are shown in Fig. 2, together with a third line $B C$ that is perpendicular to $A B$, thus forming a right triangle $A B C$. In order that the angle A shall be 34° 31′, it is necessary that the sides $A B$ and $B C$, denoted by c and a, bear a certain ratio to each other. According to Art. **4**, $\frac{a}{c}=\tan A$. But, the angle A is to be 34° 31′, and, according to the trigonometric table, the tangent of 34° 31′ is .68771. Therefore, to have an angle A of 34° 31′ $\frac{a}{c}$ must equal .68771. Make the side c exactly 2 in. long, as shown. Then, $\frac{a}{c}=\frac{a}{2}=.68771$, or $a=2\times.68771=1.37542=1\frac{3}{8}$ in. Consequently, make $B C$, or the side a, exactly $1\frac{3}{8}$ in. high and draw it at right angles to $A B$.

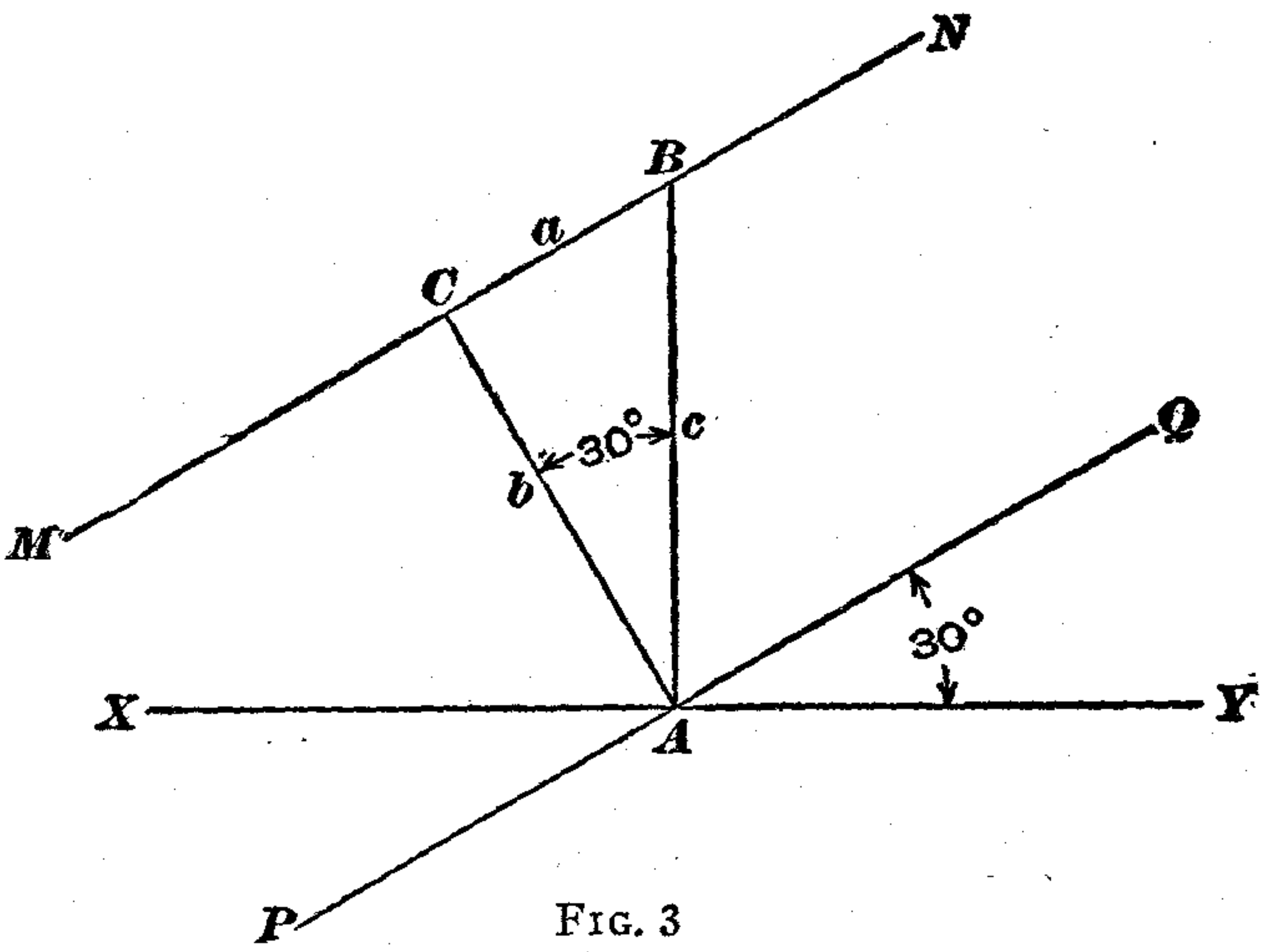

Fig. 3

Then the line $A C$ connecting A and C will make an angle of 34° 31′ with $A B$, as required. Ans.

Example 2.—In Fig. 3, $M N$ and $P Q$ are two parallel lines that slope at an angle of 30° to the horizontal line $X Y$. The distance between them, measured on the line AB, at right angles to XY, is 1.248 inches. Find the distance between them along a line AC that is at right angles to both of them.

SOLUTION.—As the line AC is perpendicular to both parallel lines, it forms a right angle ACB with the line MN; therefore, the triangle ACB is a right triangle with a right angle at C. The angle between AB and AC is 30°, because it is equal to the angle QAY, which is 30°. This is proved as follows: The angles BAY and CAQ are both right angles, because AB is perpendicular to XY and AC is perpendicular to PQ. The angle $BAY=90°=BAQ+QAY=BAQ+30°$. If $BAQ+30°=90°$, then $BAQ=90°-30°=60°$. But, $BAQ+CAB=CAQ=90°$, or $60°+CAB=90°$, from which $CAB=90°-60°=30°$. Now, as the angle A in the right triangle ABC is 30°, $\frac{b}{c}=\frac{\text{side adjacent}}{\text{hypotenuse}}=\cos A=\cos 30°$. But, $c=1.248$ in.; and from the table of trigonometric functions, $\cos 30°=.86603$. Therefore,

$$\frac{b}{1.248}=.86603$$

from which $b=1.248\times .86603=1.081$ in. Ans.

EXAMPLE 3.—A wooden brace a, Fig. 4, is to be set against a vertical post b to support a horizontal arm c. The distance from the post to the outer toe of the brace is to be 30 inches and from the arm to the lower toe of the brace is to be 48 inches. Find the angles m and n to which the ends of the brace must be cut so as to fit.

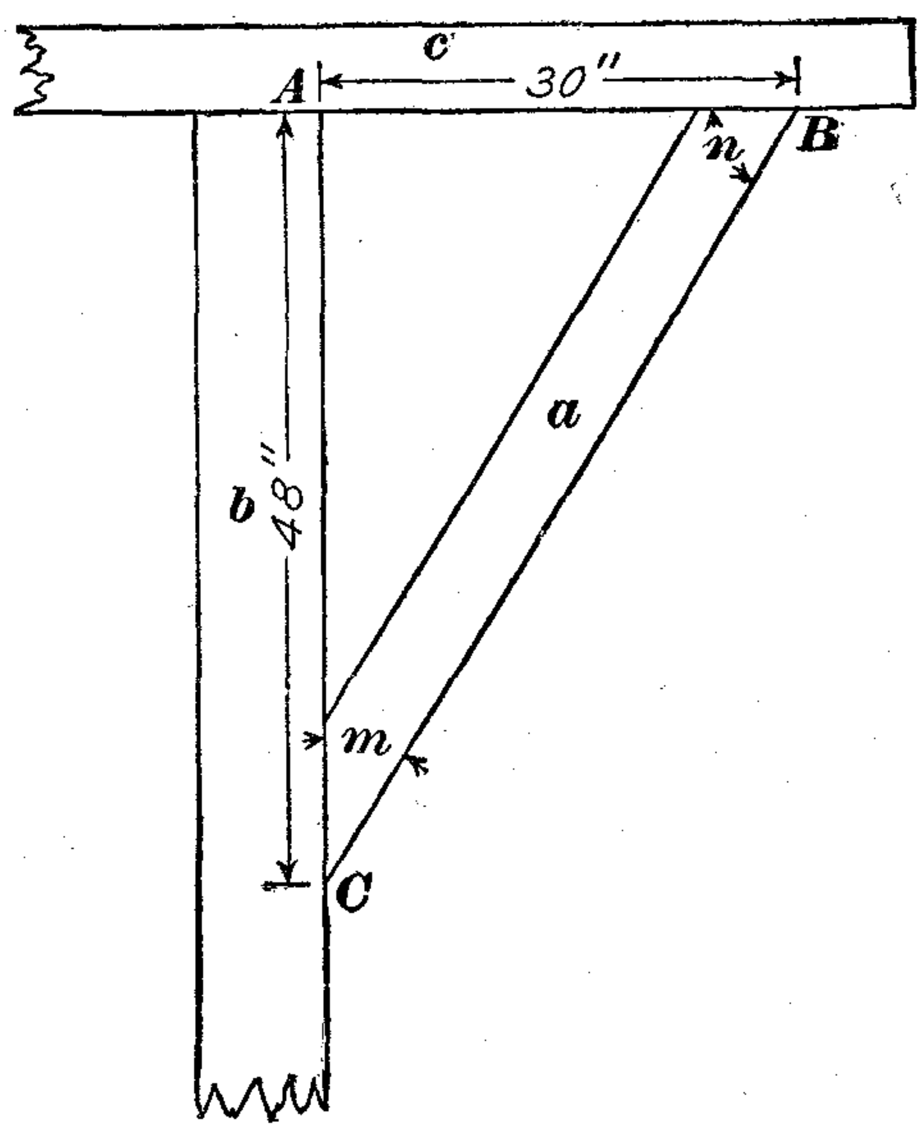

FIG. 4

SOLUTION.—The triangle ABC formed by the brace, the arm, and the post is a right triangle in which the lengths of the two sides are known. Considering the angle m, it follows from Art. **4** that $\tan m=\frac{\text{side opposite}}{\text{side adjacent}}=\frac{30}{48}=.625$. From the table of natural functions, the angle whose tangent is .625 is found to be 32°. Therefore, the lower end of the brace must be cut to an angle m of 32°. Similarly, considering the angle n at the top, $\tan n=\frac{48}{30}=1.6$. From the table, the angle whose tangent is 1.6 is found to be 58°. Therefore, the angle n must be made 58°. Ans.

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EXAMPLE 4.—If one end of a board is raised 2 feet, as shown in Fig. 5, and the board then makes an angle of 15° with the ground, find its length.

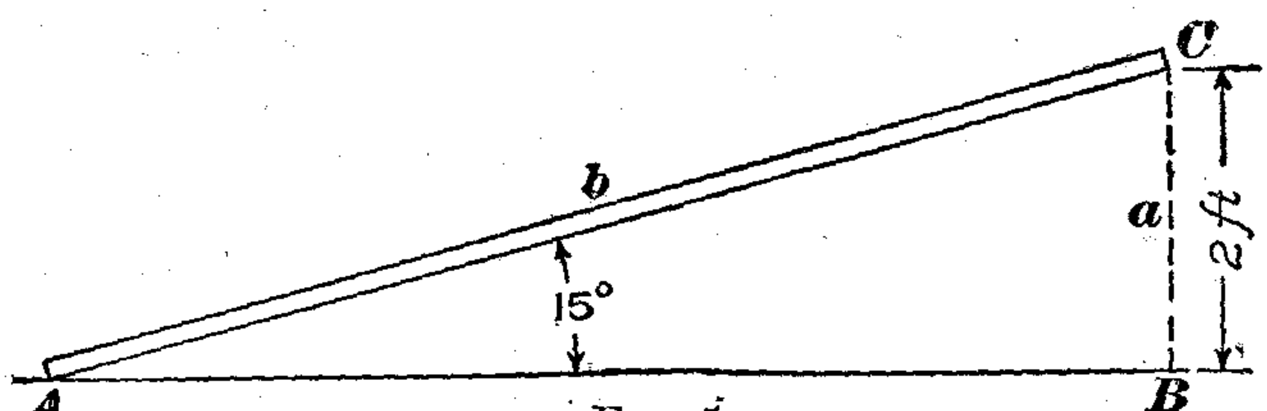

FIG. 5

SOLUTION.—The length of the board is AC, or the hypotenuse of a right triangle whose acute angle at A is 15° and whose side a is 2 ft. Let b represent the length of the hypotenuse. Then, $\frac{a}{b}=\sin A$, or $\frac{2}{b}=\sin 15°$. The sine of 15°, according to the table, is .25882. Therefore, $\frac{2}{b}=.25882$, or $.25882\,b=2$, from which

$$b=\frac{2}{.25882}=7.73 \text{ ft. Ans.}$$

EXAMPLE 5.—A ladder 30 feet long, leaning against a vertical wall, as shown in Fig. 6, makes an angle of 72° with the ground. (*a*) How far from the wall is the foot of the ladder? (*b*) To what height on the wall does the ladder reach?

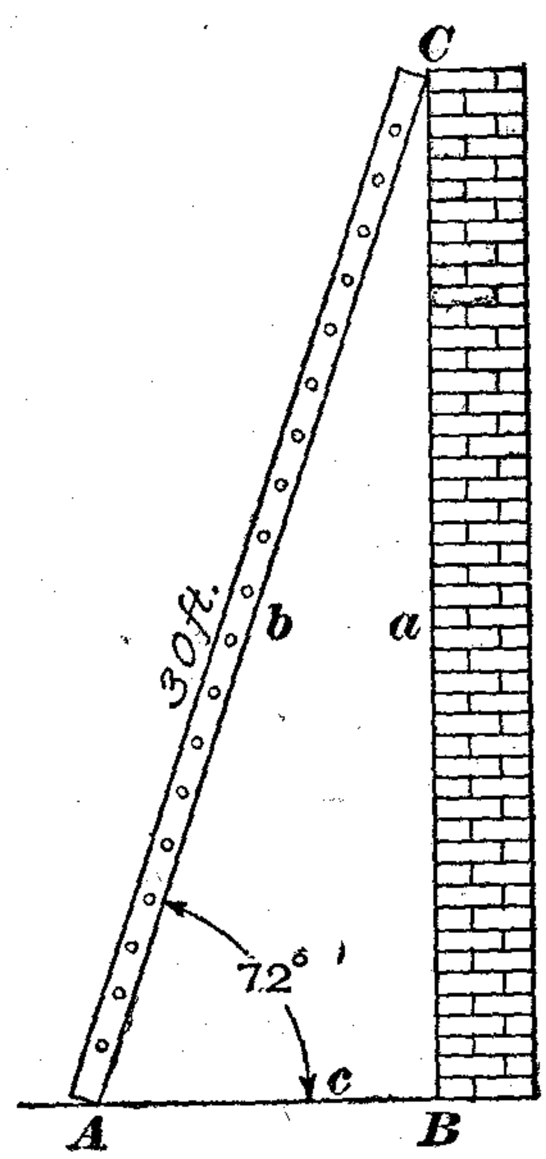

FIG. 6

SOLUTION.—(*a*) Let c represent the distance from the foot of the ladder to the wall, A the angle between the ladder and the ground, and b the length of the ladder. Then, $A=72°$, $b=30$ ft., and c is to be found. By the principles in Art. **8**, $\frac{c}{b}=\cos A$, or $c=b\times\cos A$. Then,

$$c=30\times\cos 72°=30\times.30902=9.27 \text{ ft.,}$$
or 9 ft. 3¼ in. Ans.

(*b*) Let a represent the height to which the ladder reaches. Then, according to Art. **7**, $\frac{a}{b}=\sin A$, or $a=b\times\sin A$. Then,

$$a=30\times\sin 72°=30\times.95106=28.53 \text{ ft.,}$$
or 28 ft. 6⅜ in. Ans.

EXAMPLE 6.—A building, as in Fig. 7, is 16 feet wide, and the rafters join at a point 6 feet above the top of the upper story. What is the angle of slope of the rafters?

SOLUTION.—The angle of slope is the angle at A, between the rafter and the horizontal. The rafter forms the hypotenuse of a right triangle whose base c is half the width of the building, or 8 ft., and whose height a is 6 ft. Then, $\tan A=\frac{a}{c}=\frac{6}{8}=.75$. According to the table the