# CUBE ROOT

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## EXTRACTION OF CUBE ROOT

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### METHODS AVAILABLE

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#### INTRODUCTION

**1. Definition.**—The **cube root** of a number is one of the three equal factors of that number. For example, 4 is the cube root of 64, because $4\times4\times4=64$; that is, 4 is one of three equal factors that, when multiplied together, produce 64. Similarly, 3 is the cube root of 27, because $3\times3\times3=27$. Again, 2 is the cube root of 8, because $2\times2\times2=8$; 6 is the cube root of 216, because $6\times6\times6=216$; and so on.

**2. Indicating Cube Root.**—The cube root of any number is indicated by writing the radical sign ($\sqrt{\ }$) in front of it, the vinculum ($\overline{\quad}$) over it, and the index figure $^3$ above and to the left of the radical sign. For example, $\sqrt[3]{125}$ indicates the cube root of 125; $\sqrt[3]{15.625}$ indicates the cube root of 15.625; $\sqrt[3]{\frac{27}{343}}$ indicates the cube root of $\frac{27}{343}$; and so on.

**3. Extracting Cube Root.**—The operation of finding the cube root of a number is called the *extraction* of the cube root; that is, the cube root of a number is extracted when one of the three equal factors of that number is found. The extraction of the cube roots of numbers must frequently be done in the solution of certain kinds of problems that are met with in engineering work. For this reason it is necessary that instruction be given as to how to find the cube roots of numbers.

**4. Methods of Finding Cube Root.**—There are a number of different ways in which the cube root of a number may be found, but in this Section only three methods will be described. These will be called (*a*) the trial-and-error method, (*b*) the table method, and (*c*) the exact method, and they will be described in detail in later articles.

**5. Significant Figures.**—In any number, the figures beginning with the first digit at the left and ending with the last digit at the right, are called the **significant figures** of the number. Thus, the number 405,800 has the four significant figures 4, 0, 5, 8; and the number .000090067 has the five significant figures 9, 0, 0, 6, and 7. A cipher is not a digit. Consequently, if a number begins or ends with ciphers, they are not considered in determining the significant figures of that number. But if ciphers occur between digits, as in the numbers 2,807 and 21.03, they are then considered as significant figures.

The part of a number consisting of its significant figures is called the **significant part** of the number. Thus, in the number 28,070, the significant part is 2807; in the number .00812, the significant part is 812; and in the number 170.3, the significant part is 1703.

The significant figures or the significant part of a number consists of the figures, in their proper order, from the first digit at the left to the last digit at the right, without regard to the position of the decimal point. Hence, *all numbers that differ only in the position of the decimal point have the same significant part*. For example, .002103, 210.3, 21,030, and 210,300 have the same significant figures 2, 1, 0, and 3, and the same significant part 2103.

**6. Exact and Approximate Cube Roots.**—Only a comparatively few numbers can be separated into exactly equal factors. For example, considering all the numbers from 1 to 1,000, there are only ten of which the exact cube root can be found, namely, 1, 8, 27, 64, 125, 216, 343, 512, 729, and 1,000.

All the other numbers between 1 and 1,000 have only approximate cube roots; that is, their cube roots cannot be found exactly, because those roots will contain unending decimals. For example, suppose that the cube root of 50 is to be found. There are no three equal numbers that, if multiplied together,

## TABLE I

### PERFECT CUBES AND EXACT CUBE ROOTS

| Number | Cube Root | Number | Cube Root | Number | Cube Root | Number | Cube Root |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 17,576 | 26 | 132,651 | 51 | 438,976 | 76 |
| 8 | 2 | 19,683 | 27 | 140,608 | 52 | 456,533 | 77 |
| 27 | 3 | 21,952 | 28 | 148,877 | 53 | 474,552 | 78 |
| 64 | 4 | 24,389 | 29 | 157,464 | 54 | 493,039 | 79 |
| 125 | 5 | 27,000 | 30 | 166,375 | 55 | 512,000 | 80 |
| 216 | 6 | 29,791 | 31 | 175,616 | 56 | 531,441 | 81 |
| 343 | 7 | 32,768 | 32 | 185,193 | 57 | 551,368 | 82 |
| 512 | 8 | 35,937 | 33 | 195,112 | 58 | 571,787 | 83 |
| 729 | 9 | 39,304 | 34 | 205,379 | 59 | 592,704 | 84 |
| 1,000 | 10 | 42,875 | 35 | 216,000 | 60 | 614,125 | 85 |
| 1,331 | 11 | 46,656 | 36 | 226,981 | 61 | 636,056 | 86 |
| 1,728 | 12 | 50,653 | 37 | 238,328 | 62 | 658,503 | 87 |
| 2,197 | 13 | 54,872 | 38 | 250,047 | 63 | 681,472 | 88 |
| 2,744 | 14 | 59,319 | 39 | 262,144 | 64 | 704,969 | 89 |
| 3,375 | 15 | 64,000 | 40 | 274,625 | 65 | 729,000 | 90 |
| 4,096 | 16 | 68,921 | 41 | 287,496 | 66 | 753,571 | 91 |
| 4,913 | 17 | 74,088 | 42 | 300,763 | 67 | 778,688 | 92 |
| 5,832 | 18 | 79,507 | 43 | 314,432 | 68 | 804,357 | 93 |
| 6,859 | 19 | 85,184 | 44 | 328,509 | 69 | 830,584 | 94 |
| 8,000 | 20 | 91,125 | 45 | 343,000 | 70 | 857,375 | 95 |
| 9,261 | 21 | 97,336 | 46 | 357,911 | 71 | 884,736 | 96 |
| 10,648 | 22 | 103,823 | 47 | 373,248 | 72 | 912,673 | 97 |
| 12,167 | 23 | 110,592 | 48 | 389,017 | 73 | 941,192 | 98 |
| 13,824 | 24 | 117,649 | 49 | 405,224 | 74 | 970,299 | 99 |
| 15,625 | 25 | 125,000 | 50 | 421,875 | 75 | 1,000,000 | 100 |

will produce 50; but 3.6 is an approximate cube root of 50, because $3.6 \times 3.6 \times 3.6 = 46.656$. If the root is carried to another decimal place, or made 3.68, the result is even better, because $3.68 \times 3.68 \times 3.68 = 49.836032$, which is very close to 50. If another decimal place is added and the root is made 3.684, the accuracy is still further increased, because $3.684 \times 3.684 \times 3.684 = 49.998717504$. But no matter to how many decimal places the root may be extended, the cube of that root will never be exactly 50, although it will approach nearer and nearer to 50. If an exact root cannot be found, therefore, the approximate root should be carried to as many significant figures as the accuracy of the solution demands.

**7.** Table I shows all the numbers between 1 and 1,000,000, inclusive, that have exact cube roots, these roots being the whole numbers from 1 to 100, inclusive. The larger numbers are the cubes of the corresponding smaller numbers, and any number that is a perfect cube has an exact cube root.

**8. Preliminary Steps in Finding Cube Root.**— If the cube root of a number is to be found, the first step is to point off the number into periods of three figures, beginning with the units place if the number is a whole number, and at the decimal point if the number is a decimal. If it is a mixed number, the pointing off is begun at the decimal point and is carried to the right and to the left. These instructions may be made clear by illustrative examples. Suppose, for example, that the cube root of 405,224 is to be found. The number is pointed off into periods, or parts, containing three figures, beginning at the right, or at the units place. When thus pointed off, which may be indicated by prime marks ′ or by commas, the number becomes 405′224. The reason for pointing off in this way is to determine how many figures there are in the root. For every period, or part, there will be one figure in the root; and as 405′224 contains two such periods, the cube root consists of two figures. Table I shows that the cube root of 405,224 is 74, which consists of two figures.

**9.** It may happen that, when the number whose cube root is to be found is pointed off, the first period at the left will contain one or two figures instead of three. In such a case, the one or two figures are considered as a full period. For example, suppose that the number whose cube root is to be found is 91,125. When pointed off it becomes 91′125, and as the first two figures constitute a period, there are two periods in the number, which indicates that the cube root contains two figures. Table I proves the correctness of this conclusion, as the cube root of 91,125 is 45.

If the number 5,832 is pointed off, it becomes 5′832, in which the first period contains but one figure. However, as there are two periods, the cube root contains two figures. Reference to Table I shows this to be the case, as the cube root of 5,832 is 18.

**10.** If the number whose cube root is to be found is a decimal, it is divided into periods of three figures each, *beginning at the decimal point*. In case the last period at the right contains only one or two figures, ciphers must be annexed until the period has three figures. The cube root then contains as many figures, following the decimal point, as there are periods in the original number. Annexing ciphers to the right of a decimal, as explained in a preceding Section, does not alter the value of the decimal in any way. It is done in this connection simply to fill out the period that contains less than three figures. For example, if the cube root of .14625 were required, it would be pointed off thus, .146′25, and as the last period has only two figures, a cipher would be annexed, making .146′250. As this number contains two periods, the cube root of it will be a decimal and will contain two figures, one for each period; but as the number .14625 is not a perfect cube, its cube root will not be exact and so the root may be carried to any desired number of decimal places, according to the accuracy required. This is done by annexing extra periods of three ciphers each.

**11.** If the cube root of a mixed number is to be found, the number is divided into periods of three figures each, beginning at the decimal point and proceeding in each direction. If the last period at the *right* has less than three figures, ciphers are annexed as in the case of a decimal number. For example, suppose that 1,274.4285 is the number to be considered. When pointed off it becomes 1′274.428′5, and ciphers must be annexed to the last period at the right, giving 1′274.428′500. As there are four periods, the root will contain four figures, two preceding the decimal point and two following it. In the number just considered, the decimal point serves to separate the second and third periods, and no other mark of separation is needed.

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### TRIAL-AND-ERROR METHOD

**12. Roots of Whole Numbers and Decimals.**—In solving practical problems that require the finding of cube root, it is usually not necessary to obtain the root to more than four significant figures, and in most instances three significant figures will be enough. For such approximate calculations, therefore, a simple way of finding the cube root of a number is by trial and error. As explained in Art. **1**, the cube root of a number is one of the three equal factors that, when multiplied together, will produce the number. The trial-and-error method, therefore, consists simply in assuming a root and cubing it. If the product is greater than the number whose root is to be found, the assumed root is made smaller and the cubing operation is repeated. By making a number of trials in this way, eventually a root is found that, when cubed, will give a product very nearly equal to the given number. That root is then taken as the cube root of the number. The method may be illustrated most clearly by examples and their solutions.

EXAMPLE 1.—Find the cube root of 4,913 by the trial-and-error method.

SOLUTION.—The number is first pointed off into periods, as explained in Art. **8**, which gives 4'913. As there are two periods, the cube root contains two figures. The first period consists of one figure, 4. Now, the first figure of the root cannot be 2, because $2\times2\times2=8$, which is much greater than 4. The smallest root of two figures, beginning with 2, is 20, and $20^3=20\times20\times20=8{,}000$, which is considerably greater than the given number, 4,913. So it is plain that the root must be less than 20. Suppose that 18 is assumed; then, $18\times18\times18=5{,}832$, which is also somewhat larger than 4,913, showing that the root must be less than 18. Hence, 17 is chosen; then $17\times17\times17=4{,}913$, the given number. Therefore, 17 is the exact cube root of 4,913. Ans.

EXAMPLE 2.—By the trial-and-error method find the cube root of 12,764.

SOLUTION.—When pointed off properly, the number becomes 12'764. Reference to Table I shows that the first figure of the root must be 2, because $2^3=8$, which is less than 12, the first period, whereas $3^3=27$, which is greater than the first period. If 12'764 has an exact root, there must be two figures in it, because the number has two periods, and it has just been shown that the first figure is 2. So, let 22 be chosen as a trial root; then, $22\times22\times22=10{,}648$, which is less than 12,764, showing that the root must be greater than 22. Let 23 be chosen next; then, $23\times23\times23=12{,}167$. This is also less than 12,764. So let 24 be chosen; then, $24\times24\times24=13{,}824$. This is considerably larger than the given number, 12,764, and indicates that the cube root cannot be 24. But it was also found that 23 was a little too small. Therefore, the root must be greater than 23 and less than 24. So, take 23.4 as a trial root; then, $23.4\times23.4\times23.4=12{,}812.904$. This is also larger than 12,764, showing that the root must be less than 23.4. Let 23.38 be chosen; then, $23.38\times23.38\times23.38=12{,}780.078472$, which is so close to 12,764 that it may be considered approximately equal to it. Then, 23.38 is the cube root of 12,764, very nearly. Ans.

EXAMPLE 3.—What is the value of $\sqrt[3]{.0837}$ by the trial-and-error method?

SOLUTION.—The number is first pointed off, and ciphers are annexed to complete the last period, giving .083'700. The given number is wholly decimal and so the root will be wholly decimal. As there are two periods, there will be two figures following the decimal point in the root, if the root is exact, and more if the root is not exact. The first period contains the number 83. Reference to Table I shows that the first figure of the root must be 4, because $4^3=64$, which is less than 83, whereas $5^3=125$, which is greater than 83. Hence, it is known that the root must be a number greater than .40. Let .45 be assumed; then, $.45\times.45\times.45=.091125$, which is greater than .0837, showing that .45 is too large. So .44 is chosen as a trial value; then, $.44\times.44\times.44=.085184$, which is still too large. The root must therefore be less than .44. So .43 is assumed; then, $.43\times.43\times.43=.079507$, which is smaller than .0837, indicating that the root must be greater than .43. Therefore, it is known that the root lies between .43 and .44. Let .437 be tried; then, $.437\times.437\times.437=.083453453$, or .0835, very nearly. This is so close to .0837 that, for all practical purposes, it may be said that $\sqrt[3]{.0837}=.437$. Ans.

EXAMPLE 4.—Find the value of $\sqrt[3]{442{,}800}$ to four significant figures by trial and error.

SOLUTION.—When pointed off, the number becomes 442'800. The first figure of the root must be 7, because $7^3=343$, which is less than 442, the first period, whereas $8^3=512$, which is larger than 442. As the given number has two periods, the root will have two figures preceding the decimal point. Let 76 be chosen for the first trial; then, $76 \times 76 \times 76 = 438,976$, which is less than 442,800, indicating that the root must be greater than 76. If 77 is tried, it is found to be too large, because $77^3 = 456,533$, which exceeds 442,800. Therefore, the root lies somewhere between 76 and 77. As 438,976 is closer to 442,800 than is 456,533, the root is closer to 76 than to 77. So, let 76.3 be chosen as a trial value; then, $76.3^3 = 444,194.947$, which is larger than 442,800, indicating that 76.3 is too large. If 76.2 is tried, it is found that $76.2^3 = 442,450.728$, which is a trifle smaller than 442,800, but very close to it. It is concluded, then, that the root is between 76.2 and 76.3, but not much greater than 76.2. Take 76.21 as a trial value, and $76.21^3 = 442,625$, very nearly. This is still less than the given number, so 76.22 is tried as the root. Then, $76.22^3 = 442,799$, which is so nearly equal to 442,800 that 76.22 may be considered as the required root, to four significant figures. Ans.

**13. Roots of Numbers Containing Fractions.**—If the number is a fraction or if it contains a fraction, the simplest way of treating it is to reduce the fraction to a decimal and then proceed as in the preceding examples. For example, suppose that the cube root of $\frac{3}{8}$ is to be found by trial and error. Reduce the fraction to a decimal, thus: $\frac{3}{8} = 3 \div 8 = .375$. Then, the cube root of .375 is found by the method already explained, and the result thus obtained is the root of $\frac{3}{8}$. If the cube root of a number like $17\frac{26}{47}$ is to be found, the fractional part is reduced to a decimal; thus, $\frac{26}{47} = 26 \div 47 = .5532$. The entire number then becomes 17.5532, and the root is found in the manner already explained. The following examples will serve to make the foregoing instructions clearer:

Example 1.—Find by trial and error the cube root of $\frac{5}{8}$, to three significant figures.

Solution.—The fraction $\frac{5}{8}$, reduced to a decimal, becomes .625. As .625 is wholly a decimal the cube root will be wholly a decimal. The first period of .625 contains three figures, the remaining periods consisting of ciphers; that is, the number when pointed off may be written .625′000′000. The largest number whose cube is less than .625 is .8, because $.8^3 = .512$, whereas $.9^3 = .729$. Therefore, the first figure of the root is 8 and the root lies somewhere between .8 and .9. Choose .85 as a trial root; then, $.85^3 = .614125$, which is less than .625. So, try .86 as the root; then, $.86^3 = .636056$, which is too large, as it exceeds .625. The root of .625 therefore lies between .85 and .86. Choose .855 as a trial root; then, $.855^3 = .625026375$. which is so close to .625 that no further calculations are necessary, and .855 may be taken as the cube root of $\frac{5}{8}$=.625. Ans.

EXAMPLE 2.—Find the value of $\sqrt[3]{470\frac{3}{4}}$ to three significant figures by trial and error.

SOLUTION.—First reduce $\frac{3}{4}$ to a decimal; thus, $\frac{3}{4}=3\div4=.75$. The number whose cube root is to be found is then 470.75. When this number is pointed off, it becomes 470.750, showing that the root has one figure before the decimal point. The largest number whose cube is less than 470 is 7, because $7^3=343$, whereas $8^3=512$. So the first figure of the root is 7 and the required root is between 7 and 8. Let 7.5 be assumed; then, $7.5^3=421.875$, which is less than 470.75. So 7.6 is tried; then, $7.6^3=438.976$, which is still too small. Next, 7.7 is tried; then, $7.7^3=456.533$, which also is less than 470.75. Therefore, 7.8 is tried; then, $7.8^3=474.552$, which is too large. It is now seen that the root is greater than 7.7 but less than 7.8. As the cube of 7.8 is 474.552, which is close to the given number 470.75, it is plain that the root must be close to 7.8, but less than 7.8. So, 7.78 is tried; then, $7.78^3=470.911$, which is so close to 470.75 that no further trials are necessary, and the required root is taken as 7.78. Ans.

## EXAMPLES FOR PRACTICE

Find the cube roots of the following numbers, by the trial-and-error method, to three significant figures:

| | | | |
|---|---|---|---|
| (*a*) | 18,610 | Ans. (*a*) | 26.5 |
| (*b*) | .3505 | (*b*) | .705 |
| (*c*) | 160,000,000 | (*c*) | 543 |
| (*d*) | 54.5 | (*d*) | 3.79 |
| (*e*) | $\frac{1}{2}$ | (*e*) | .794 |
| (*f*) | $12\frac{5}{16}$ | (*f*) | 2.31 |

## TABLE METHOD

**14.** In Table I are given the cubes of all whole numbers from 1 to 100, inclusive; likewise, the table gives the cube roots of all numbers between 1 and 1,000,000 that have exact roots. By means of this table it is possible to find directly the first two significant figures in the cube root of any number and to find the third significant figure by a simple calculation. It will be observed that the numbers in the table are all whole numbers, and that none contains more than seven figures. Therefore, to use the table for finding cube roots, the numbers whose roots are to be found must be altered in such a way as to bring them within the range of the table. The method of doing this will now be described.

**15.** As already explained, digits only are considered in determining the significant figures of a number; therefore, 5, 50, .5, and .05 have the same significant figure, 5. The cubes of these numbers are 125, 125,000, .125, and .000125, respectively, and thus it will be seen that these cubes also have the same significant figures, 1, 2, and 5, and the same significant part, 125. The numbers 125, 125,000, .125, and .000125, if pointed off preparatory to extracting the cube roots, become 125, 125′000, .125, and .000′125. It will be seen at once that each of these contains the same significant part, 125, and that these three significant figures form a single period in each number. The only difference among the numbers is in the position of the decimal point. Consequently, their cube roots will have the same significant figure, 5, but the position of the decimal point will be different. Thus, as 125 is a whole number of one period, its root is a single figure, 5; as 125′000 has two periods, its root has two figures, 50; as .125 is a decimal and comprises one period, its root is a decimal of one figure, or .5; and as .000′125 is a two-period decimal, its root is a decimal of two figures, or .05, because, when a decimal begins with ciphers, there is a cipher in the root, following the decimal point, for each complete period of *three* ciphers directly following the decimal point in the number.

**16.** The principle explained in the preceding article is important and should be thoroughly understood, as it forms the basis for the use of Table I for finding cube root. Suppose, for example, that the cube root of .004096 is to be found. The table contains no decimals whatever. But, if .004096 is pointed off, it becomes .004′096, which is a number of two periods with a single figure, 4, in the first period. The significant part of the number is 4096. In the table can be found 4,096, which also contains two periods, with the figure 4 alone in the first period, and which has the same significant figures. Therefore, 4,096 and .004096 will have the same significant figures in their roots, the only difference being in the location of the decimal point. The cube root of 4,096 is 16, according to the table. The cube root of .004096 contains these same significant figures, 16, but as .004′096 has two periods and is wholly a decimal, its root must have two figures and be wholly a decimal. Therefore, the cube root of .004096 must be .16.

**17.** It has been shown that after a number has been pointed off into full periods of three figures each, by the method explained in Arts. **8** to **11**, the position of the decimal point does not affect the figures of the root. Therefore, after the periods have been determined the number may be considered as a whole number while the figures of the root are being found. If this whole number does not consist of two periods it must be altered, because no number having more than two periods is given in the table and because accurate results cannot usually be obtained by using only one period.

If the number has only one period, three zeros are annexed to form the second period. Thus, if 3 is the number whose cube root is to be found, it is necessary to annex three ciphers to form the second period; the altered number is 3′000. If the number contains more than two periods, ignore the periods after the second, unless the first figure of the third period is 5 or more, in which case increase the last figure in the second period by 1. If the number were 2,743,879, it would be pointed off thus, 2′743′879. As there are three periods, the last period is dropped. Since the first figure of the third period exceeds 5, the last figure of the second period is increased by 1. The altered number is 2744, and is found in the table.

**18.** If the cube root of the altered number may be found from the table directly it is merely necessary to locate the decimal point in accordance with its position in the original number; the cube root of the original number has the same significant figures as the cube root of the altered number.

It frequently happens that the altered number cannot be found in Table I. In such a case the numbers next greater and smaller than the altered number are found, and then, by the method to be illustrated in subsequent examples, the root is determined.

**19.** If the number whose root is to be found contains a fraction, reduce the fraction to an equivalent decimal, and then proceed as already explained. For example, if the cube root of $\frac{11}{16}$ is to be found, the fraction is reduced to a decimal; thus, $\frac{11}{16}=11\div16=.6875$. This decimal, when pointed off, becomes .687′500, showing that the root is wholly a decimal, with a digit immediately following the decimal point. The significant figures of the root are exactly the same as those of the cube root of 687,500, which becomes the altered number. If the cube root of $37\frac{1}{8}$ is to be found, the fraction $\frac{1}{8}$ is reduced to a decimal, becoming .125, and the number thus becomes 37.125. The altered number is 37,125, which has the same significant figures for its cube root as has 37.125.

EXAMPLE 1.—Find the value of $\sqrt[3]{12,817}$ by the use of Table I.

SOLUTION.—The given number, 12,817, contains two periods and is a whole number, so that it need not be altered. The number 12,817 does not appear in the table, but the numbers 12,167 and 13,824 appear, and 12,817 lies somewhere between them. The cube root of 12,167 is 23 and the cube root of 13,824 is 24; therefore, the cube root of 12,817 must lie between 23 and 24. It may be assumed that the desired root is at the same point between 23 and 24 as 12,817 is between 12,167 and 13,824. To determine just what this intermediate value of the root may be, the following simple calculation is made: The difference between the two numbers in the table is found, and is called the **first difference;** thus, 13,824−12,167=1,657, which is the first difference. Next the difference between the given number and the *smaller* number in the table is found, and is called the **second difference;** thus, 12,817−12,167=650, which is the second difference. The second difference is then divided by the first difference; thus, 650÷1,657=.39, or .4, nearly. This figure then becomes the third figure of the root. As the first two figures were already found to be 2 and 3, because the root is greater than 23 but less than 24, the required root is 23.4. Ans.

PROOF.—The accuracy of any result found in extracting cube root may be tested very readily by simply cubing the value found for the root. The product should be approximately equal to the given number. In the preceding example, for instance, the root was found to be 23.4. By the foregoing test, $23.4^3=23.4\times23.4\times23.4=12,813$, very nearly. As this is very close to 12,817, the given number, it is concluded that 23.4 is the cube root, correct to three significant figures.

EXAMPLE 2.—Find by the table method the cube root of .0732 to three decimal places.

SOLUTION.—When pointed off, the given number becomes .073′200. The significant figures of this number are the same as if the number were 73,200; also, the significant figures of the cube root of .073200 are the same as those of the cube root of 73,200, because each contains two periods composed of like figures. The only difference is that the cube root of .073200 is wholly a decimal, whereas the cube root of 73,200 is a mixed number in which the decimal point occurs after the second figure. All that is necessary, therefore, is to find the cube root of 73,200 from Table I and then to shift the decimal point in that root until the root is wholly a decimal. The result will then be the cube root of .0732. In the table, 73,200 lies between 68,921 and 74,088, the cube roots of which are 41 and 42, respectively. Therefore, the cube root of 73,200 is greater than 41 and less than 42. The third significant figure of the root is found as follows:

| | | | |
|---|---|---|---|
| 74,088 | | 73,200 | |
| 68,921 | | 68,921 | |
| 5,167 | *first difference* | 4,279 | *second difference* |

*Second difference* ÷ *first difference* = 4,279 ÷ 5,167 = .8

Then, the third significant figure is 8 and the cube root of 73,200 is 41.8. The cube root of .0732 has exactly the same significant figures, but as .0732 is wholly a decimal, its cube root is wholly a decimal; therefore, $\sqrt[3]{.0732} = .418$. Ans.

EXAMPLE 3.—What is the cube root of .18526?

SOLUTION.—When pointed off, the given number becomes .185′260. The cube root of this number has the same significant figures as the cube root of 185,260, but is wholly a decimal. In the table, 185,260 lies between 185,193 and 195,112, showing that the cube root of 185,260 lies between 57 and 58. The first difference is 195,112−185,193=9,919 and the second difference is 185,260−185,193=67; then, 67÷9,919=.007, nearly. Annex this to 57, and the root is 57.007 or 57.01 to four significant figures. As the cube root of .18526 has the same significant figures, but is wholly a decimal, it must be .5701. Ans.

EXAMPLE 4.—Find from the table the value of $\sqrt[3]{293,475,910}$.

SOLUTION.—When pointed off into periods, the given number becomes 293′475′910. The third period is dropped; but, as it begins with 9, the last figure of the second period is increased by 1, as explained in Art. **17.** The altered number thus becomes 293′476. This is not found in the table, but 287,496 and 300,763 are found there. Therefore the required root is greater than 66 and less than 67. The first difference is 300,763−287,496=13,267 and the second difference is 293,476 −287,496=5,980. Then, 5,980÷13,267=.45, showing that the third and fourth significant figures are 4 and 5. The cube root of 287,496 is 66, according to the table, and so the root of 293,476 is 66.45. The original number, however, is a whole number containing three periods; therefore, its root must have three figures ahead of the decimal point. Hence, the root is 664.5. Ans.

EXAMPLE 5.—What is the cube root of .0007425?

SOLUTION.—The given number, pointed off into complete periods, is .000′742′500. According to Art. **17**, this number must be changed to one containing the same significant figures and having two periods. The first period, consisting of ciphers, is therefore dropped, and the altered number is 742,500. It does not appear in the table, but 729,000 and 753,571, the cubes, respectively, of 90 and 91, are given therein. The first difference is 753,571−729,000=24,571, and the second difference is 742,500−729,000=13,500. Then, 13,500÷24,571=.5, approximately. The cube root of 729,000 is 90, and to this must be annexed the .5 just found, giving 90.5 as the approximate cube root of 742,500. The original number, .0007425, has one complete period of ciphers following the decimal point, and so the root must be a decimal having one cipher following the decimal point and in front of the first significant figure. Therefore, the required root is .0905. Ans.

EXAMPLE 6.—Find the value of $\sqrt[3]{\frac{273}{1054}}$.

SOLUTION.—First reduce the fraction to a decimal; thus, $\frac{273}{1054}$=273 ÷1,054=.259013. The problem therefore is to find the cube root of .259013. This number, when pointed off, is .259′013, and the altered number is 259,013. This altered number is not in Table I, but it lies between 250,047 and 262,144, the cube roots of which are 63 and 64, respectively. The first difference is 262,144−250,047=12,097, and the second difference is 259,013−250,047=8,966. Then, 8,966÷12,097=.7, and the cube root of 259,013 is 63.7. The cube root of .259013 has the same significant figures, but is wholly a decimal; therefore, it must be .637. Ans.

EXAMPLE 7.—What is the cube root of $3\frac{2}{3}$?

SOLUTION.—When reduced to a decimal, $\frac{2}{3}$ becomes .6666+ or .667, and the number whose root is to be found is then 3.667. The altered number is 3,667, which lies between 3,375 and 4,096 in the table; and as the roots of 3,375 and 4,096 are 15 and 16, respectively, the root of 3,667 must lie between 15 and 16. The first difference is 4,096−3,375 =721, and the second difference is 3,667−3,375=292. Then, 292÷721 =.4. So the cube root of 3,667 is 15.4. The cube root of 3.667, which has one period preceding the decimal point, has the same significant figures, with one figure preceding the decimal point; therefore, the cube root required is 1.54. Ans.

### EXAMPLES FOR PRACTICE

By the use of Table I find the cube roots of the following numbers to three significant figures:

(*a*) 860,000
(*b*) 27.27
(*c*) .09125
(*d*) 127.4
(*e*) $56\frac{7}{8}$
(*f*) $\frac{49}{64}$

Ans. (*a*) 95.1; (*b*) 3.01; (*c*) .450; (*d*) 5.03; (*e*) 3.85; (*f*) .915

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### EXACT METHOD

**20.** If the cube root of a number is to be found accurately to more than three significant figures, neither of the preceding methods can be used successfully. In such a case, an exact method is required. There are a number of such exact methods, but the one about to be described is simple and may be used for finding the cube root of any number. It is most easily explained by the solution of an example.

**21. Cube Roots of Whole Numbers.**—Suppose that it is desired to find the cube root of 11,390,625. First separate the number into periods of three figures each, commencing at the right. This gives 11′390′625. Draw a line at the right of the number and find the largest number whose cube is not greater than the left-hand period. Place this number as the first figure of the answer.

The left-hand period is 11 and the largest number whose cube will not be greater than 11 is 2, since $3^3=27$.

Place the 2 as the first figure of the answer and subtract the cube of this number from the left-hand period of the number. To the remainder thus obtained annex the next period of the number. The process thus far is as follows:

```
11'390'625(2
 8
 -
 3390
```

It is next necessary to find a *trial divisor*. This is obtained by considering the number in the answer as tens, *squaring* it, and multiplying the result by 3. 2 considered as tens is 20, which squared gives 400. 400 multiplied by 3 gives 1,200 as a trial divisor.

The trial divisor is contained 2 times in the remainder 3,390. Place the 2 as the second figure of the answer. This gives the following:

```
                    1 1'3 9 0'6 2 5 ( 2 2
                      8
                      —
3×20² = 1200   |      3 3 9 0
```

In order to obtain a *complete divisor*, it is necessary to add together three products.

The *first* product is that obtained for the trial divisor, or, in this case, 1,200.

The *second* product is obtained by considering the first figure of the answer as *tens*, and multiplying it by the *second* figure of the answer, and by 3. In the example being worked, the first figure of the answer considered as tens gives 20, which multiplied by the second figure of the answer, or 2, gives 40, which result multiplied by 3 gives 120 as the second product.

The *third* product is obtained by *squaring* the second figure of the answer, which in this case is $2^2 = 4$.

Add these three products together and the complete divisor becomes $1{,}200 + 120 + 4 = 1{,}324$. The example worked to this point is as follows:

```
                              1 1'3 9 0'6 2 5 ( 2 2
                                8
                                —
     3×20² = 1 2 0 0    |       3 3 9 0
3×(20×2)   =   1 2 0    |
        2² =       4    |
             -------    |
             1 3 2 4    |
```

Next multiply the complete divisor 1,324 by the second figure of the answer and subtract the result from the remainder previously obtained. To the result annex the next period for a *new remainder*. The process is as follows:

| | |
|---|---|
| | 1 1′3 9 0′6 2 5 ( 2 2 |
| | 8 |
| $3\times20^2 = 1200$ | $\bar{3}$ 3 9 0 |
| $3\times(20\times2) = 120$ | |
| $2^2 = 4$ | |
| 1 3 2 4 | 2 6 4 8 |
| | 7 4 2 6 2 5 |

The further processes are but repetitions of those previously described and will not need much explanation.

It is next necessary to find a trial divisor for the remainder. This is done in the same manner as in the first case, the number in the answer being considered as tens, and squared, after which it is multiplied by 3. This will be $220^2 = 48{,}400$, which, multiplied by 3, gives 145,200 for a trial divisor.

The trial divisor divided into the remainder gives 5 as the next figure of the answer.

Next, find the complete divisor by adding together the three products, taking the trial divisor as the first product, 3 times the number already in the answer considered as tens multiplied by the next figure of the answer for the second product, and the square of the third figure of the answer for the third product. Multiply the complete divisor by the third figure of the answer and subtract the result thus obtained from the remainder previously obtained. The complete process is:

| | |
|---|---|
| | 1 1′3 9 0′6 2 5 ( 2 2 5 Ans. |
| | 8 |
| $3\times20^2 = 1200$ | $\bar{3}$ 3 9 0 |
| $3\times(20\times2) = 120$ | |
| $2^2 = 4$ | |
| 1 3 2 4 | 2 6 4 8 |
| $3\times220^2 = 145200$ | 7 4 2 6 2 5 |
| $3\times(220\times5) = 3300$ | |
| $5^2 = 25$ | |
| 1 4 8 5 2 5 | 7 4 2 6 2 5 |

Since the product obtained by multiplying the complete divisor by the third term of the answer is just equal to the remainder, the process is complete, and the answer, or 225, is the cube root of the number 11,390,625.

**22. Cube Roots of Decimals.**—In extracting the cube root of a decimal, proceed as with a whole number, taking care that each period contains *three* figures. Begin the pointing off at the decimal point, going toward the right. If the last period does not contain three figures, annex ciphers until it does.

The number of decimal places in the answer will equal the number of periods in the decimal whose root is to be extracted.

EXAMPLE.—What is the cube root of .009129329?

SOLUTION.—

| | | |
|---|---|---|
| | | .009'129'329(.209 Ans. |
| | | 8 |
| $3\times20^2=$ | 1200 | 1129 |
| | | 0000 |
| $3\times200^2=$ | 120000 | 1129329 |
| $3\times(200\times9)=$ | 5400 | |
| $9^2=$ | 81 | |
| | 125481 | 1129329 |

In this particular example it will be noticed that in the first case the trial divisor 1,200 is greater than the remainder 1,129; consequently, the next figure of the answer is 0 and it is necessary to bring down the next period for a new remainder, after which the regular process is followed.

**23. Roots of Mixed Numbers.**—One example of extracting the cube root of a mixed number will be given here.

EXAMPLE.—What is the cube root of 47.832147?

SOLUTION.—

| | | |
|---|---|---|
| | | 47.832'147(3.63 Ans. |
| | | 27 |
| $3\times30^2=$ | 2700 | 20832 |
| $3\times(30\times6)=$ | 540 | |
| $6^2=$ | 36 | |
| | 3276 | 19656 |
| $3\times360^2=$ | 388800 | 1176147 |
| $3\times(360\times3)=$ | 3240 | |
| $3^2=$ | 9 | |
| | 392049 | 1176147 |