MENSURATION

Serial 1981 Edition 1

LINES, SURFACES, AND SOLIDS

LINES AND ANGLES

LINES

1. Geometry.—That branch of mathematics which deals with the relations, properties, and measurements of lines, angles, surfaces, and solids is known as **geometry.** The particular branch of geometry that treats only of the measurement of lines, angles, surfaces, and solids is known as **mensuration.**

In laying out work the mechanic will frequently make use of lines and angles in constructing various surfaces. The general property of these elements, as well as some methods of constructing them, will be explained in this Section, together with the calculation of surface areas and the contents of solids.

2. Points.—In mathematics a **point** is supposed to be without any dimensions; that is, it has no length, breadth, or thickness. In accurate work the position of a point is indicated by two lines crossing each other at the place where the point is intended to be located.

3. Lines.—From a mathematical point of view a **line** has only one dimension—*length;* it serves to connect two points. On paper a line is represented by means of a mark, that may vary in thickness, depending on the means used to make it.

A **straight line,** Fig. 1 (*a*), is one that does not change direction throughout its whole length. A straight line is frequently called a *right line.* Lines shown in illustrations are

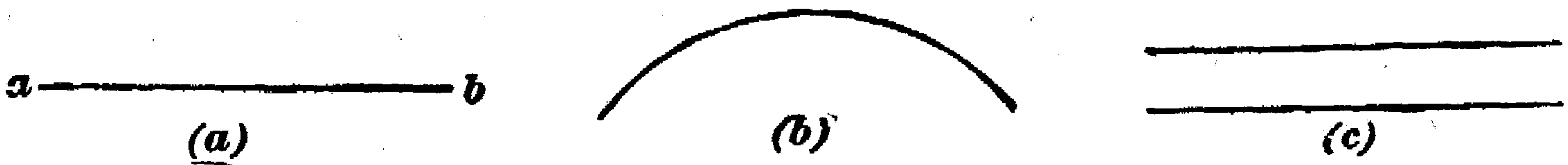

FIG. 1

usually referred to by letters at their ends, as *a b,* Fig. 1 (*a*).

A **curved line,** Fig. 1 (*b*), changes its direction at every point, no part of the line being straight.

4. Lines Named According to Relative Positions. When two straight lines are equally distant from each other throughout their length, as in Fig. 1 (*c*), they are said to be **parallel lines.** Either one of two parallel lines is said to be *parallel to* the other.

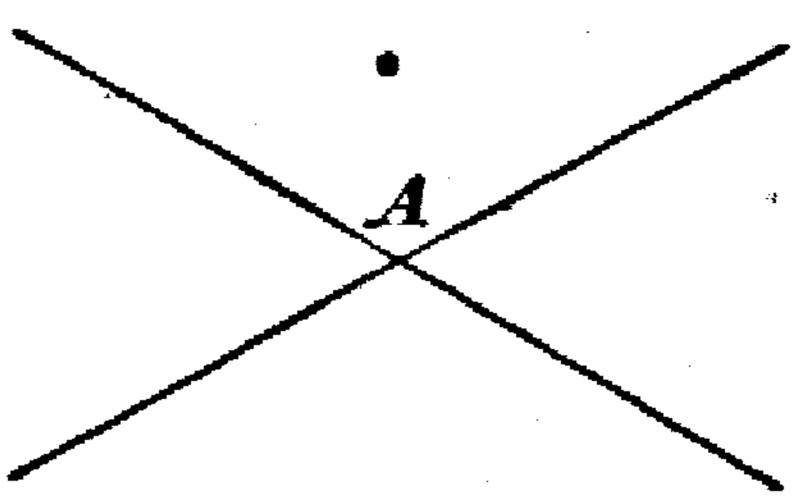

FIG. 2

5. When two lines cross or cut each other, as in Fig. 2, they are said to be **intersecting lines,** and the point *A,* at which they intersect, or cut each other, is called the **point of intersection.**

6. A line is said to be **perpendicular** to another line, when it meets that line so as not to incline toward it on either side. Thus, in Fig. 3 (*a*), the line *A B* is perpendicular to the

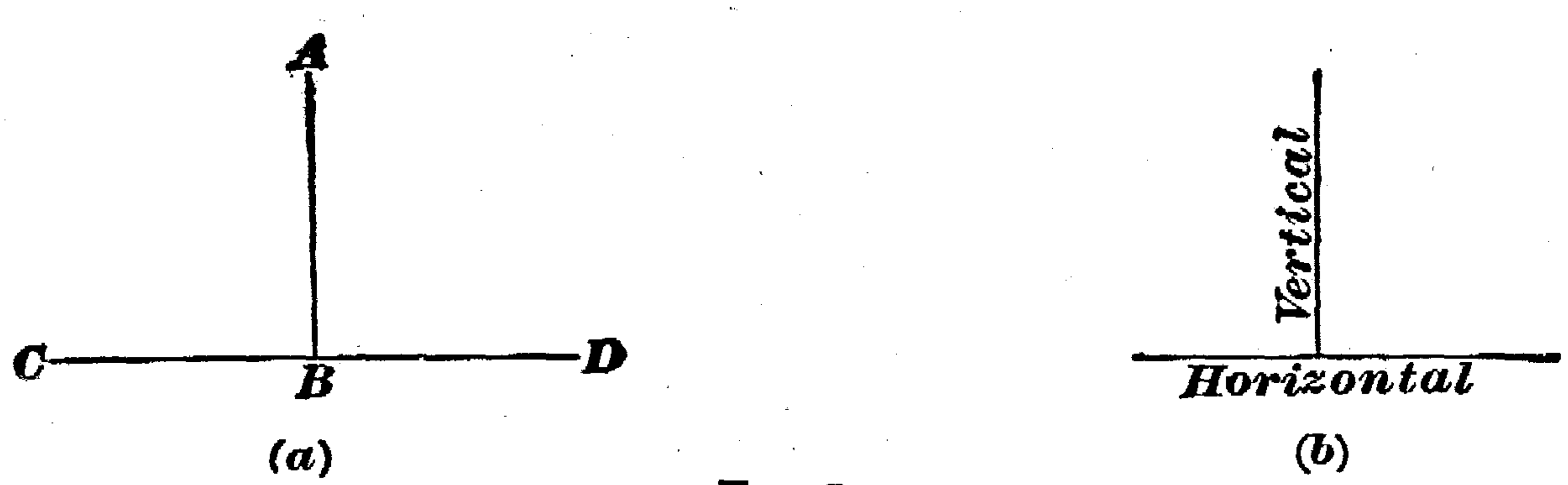

FIG. 3

line *C D*; also, *C D* is perpendicular to *A B.* Either line is also said to be **at right angles** to the other.

7. A line that is parallel to the horizon, or water level, as *C D* in Fig. 3 (*a*) or the corresponding line in Fig. 3 (*b*), is known as a **horizontal line.**

9. Drawing Perpendicular Lines.—In laying out some classes of work, it is necessary to draw one line perpendicular, or at right angles, to another; therefore, methods of doing this will be explained. In the example, Fig. 4, a line *A B* is drawn, and it is desired to draw a line perpendicular to it at the point *P*. To do this, the legs of a pair of compasses are separated to any convenient distance, the pointed leg is set at the point *P* while with the other leg, which carries a pencil, two marks are made at *C* and *D* on the line *A B*. These marks will be at equal distances from the point *P* and on opposite sides of it. The legs of the compasses are now brought farther apart, so that they will span a distance greater than *P C*, the pointed leg is set at *C*, and a short arc (or part of a circle) is struck with the pencil above *P*, as at *F*. The leg situated at *C* is now removed to *D*, with the legs of the compasses the same distance apart as before, and another arc is struck at *F*. The points at which these arcs intersect is the point required, as a line drawn from it to *P* will be perpendicular to *A B*.

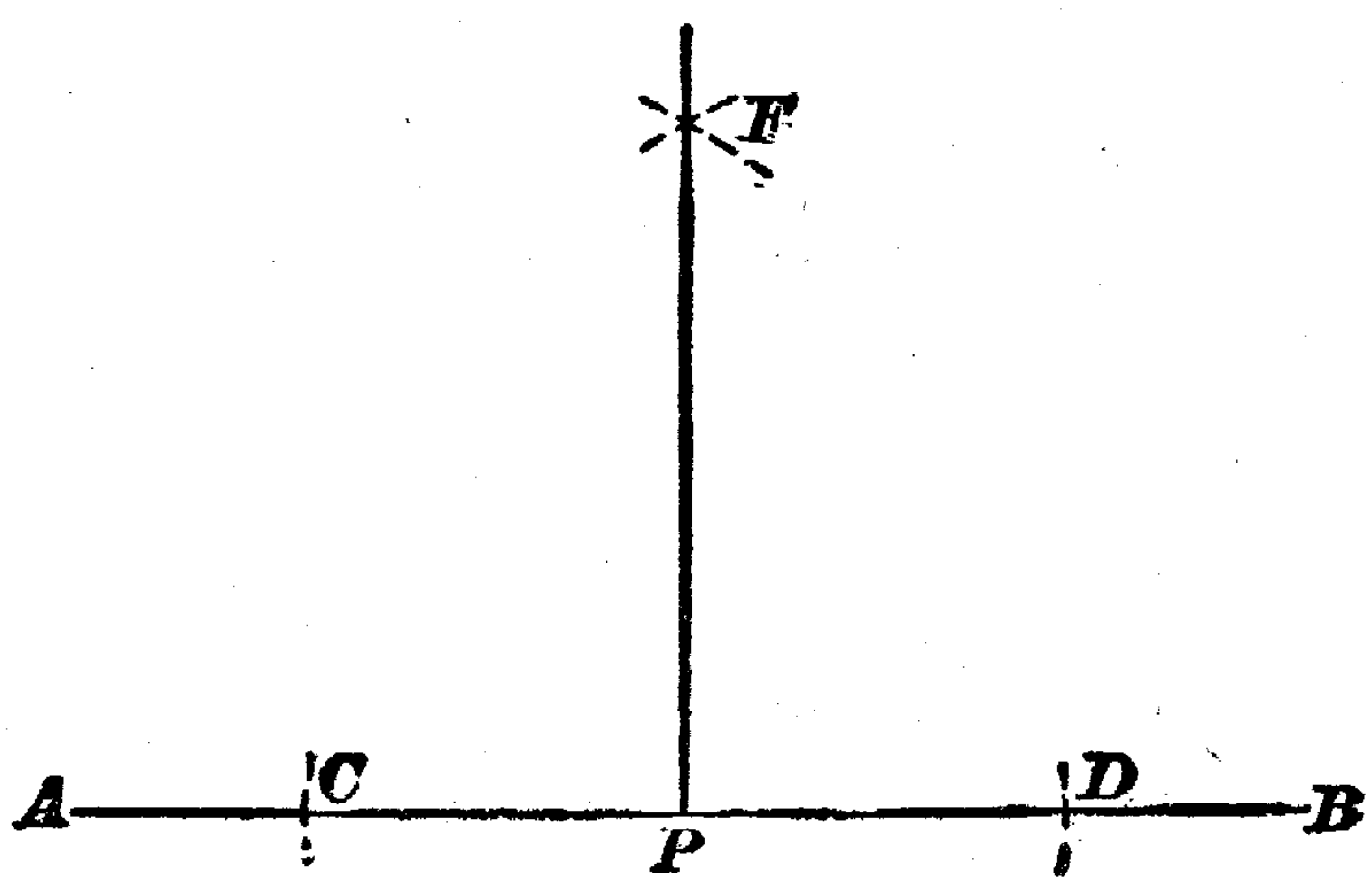

FIG. 4

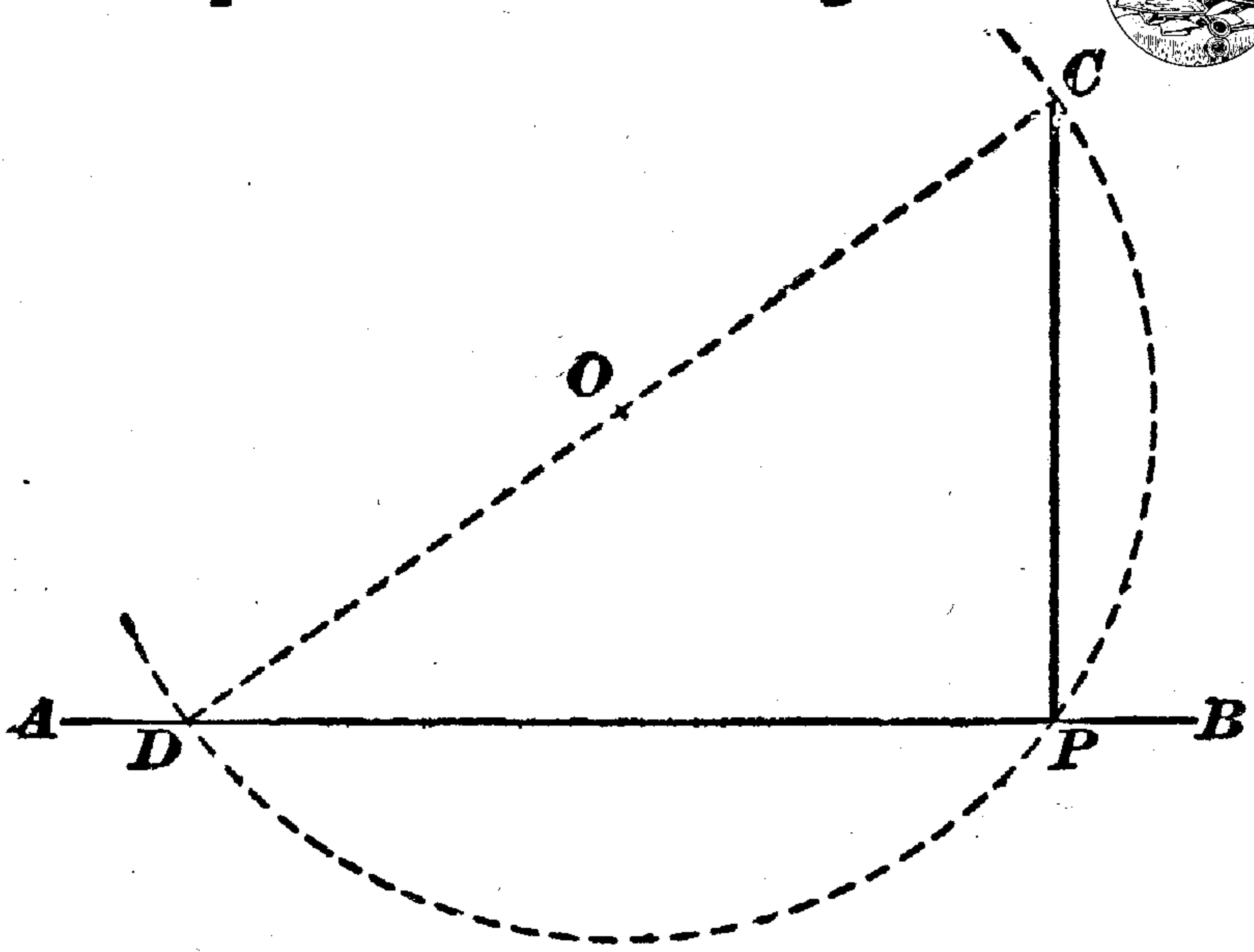

FIG. 5

10. If the point *P*, Fig. 4, at which the perpendicular is to be drawn, lies near the end of the line, as, for instance, at the edge of a piece of work, the foregoing method cannot be used. In such cases, the construction shown in Fig. 5 may be applied.

where $A B$ is the line and P the point at which the perpendicular is to be drawn. The legs of the compasses are separated to some convenient distance, one leg is placed at a point above $A B$, as at O, and with the other the greater part of a circle is drawn so as to pass through the point P, as shown by the dotted curve. This circular arc will also cross the line $A B$ at D. From the latter point a line is drawn through the center O and continued till it intersects the arc at C. A line drawn from the point C to the point P will then be perpendicular to $A B$.

11. Sometime a perpendicular has to be drawn to a line from a given point outside the line. The method to be employed in this case is shown in Fig. 6, in which $A B$ is the line and P the given point. One leg of the compasses is placed at P and with the other leg an arc is described, so that it will intersect the line $A B$ in any two points, C and D, as shown by dotted curve. One leg of the compasses is placed successively at the points C and D, and short arcs are described with the other leg at E. From their point of intersection, E, a line is drawn to P; then $E P$ is perpendicular to $A B$.

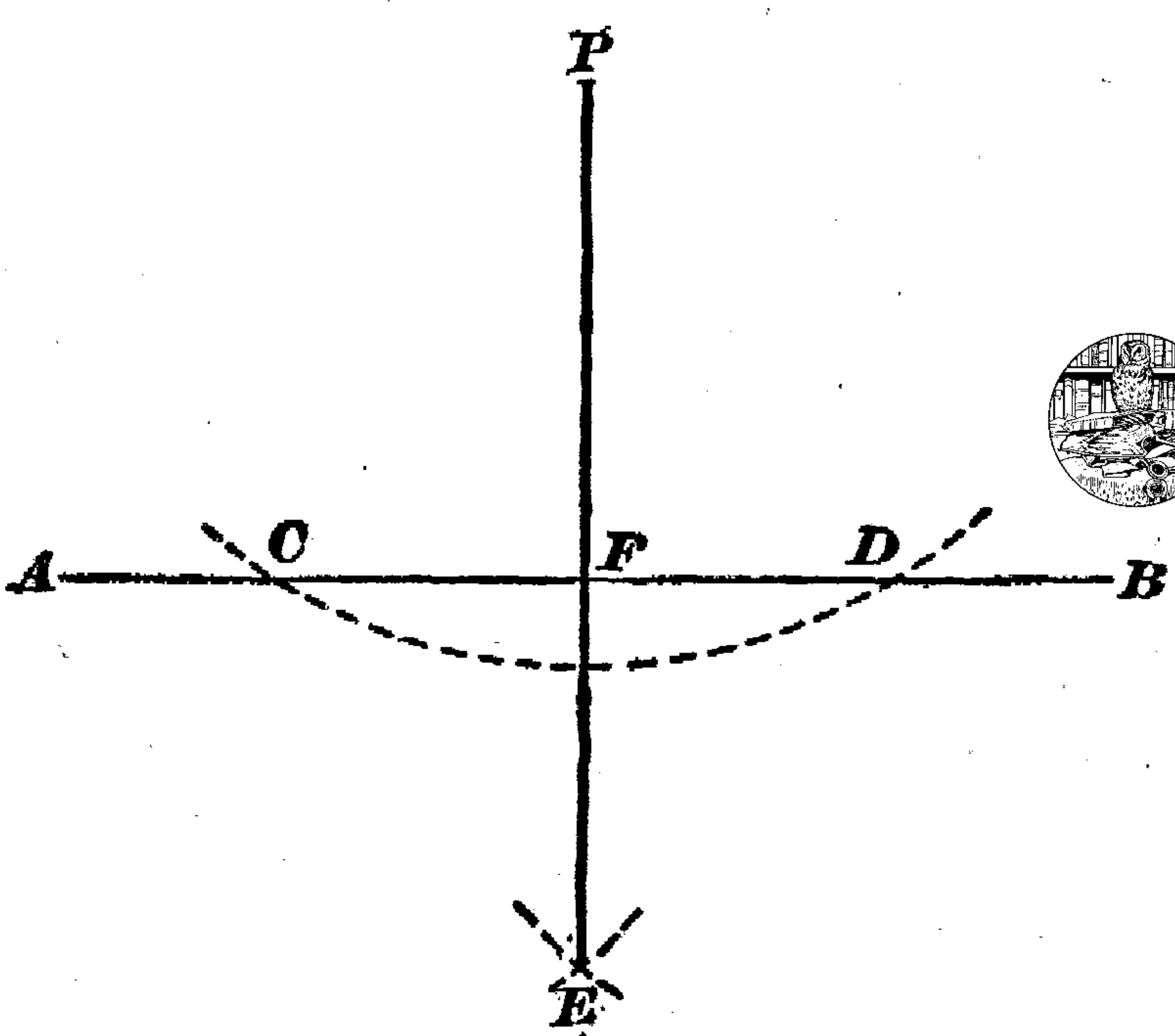

FIG. 6

A perpendicular, drawn from a point outside a straight line, is the **shortest distance** from the point to the line. Thus, in Fig. 6, the perpendicular $P F$ is the shortest distance from P to $A B$.

ANGLES

12. Definition of an Angle.—When two lines, as $a b$ and $b c$, Fig. 7, meet or intersect at a point b, the opening between the lines is known as an **angle.** The lines are known as the **sides** and the point b as the **vertex** of the angle.

An angle may also be defined as the difference in direction between two straight lines. This may be made clearer by means of a practical illustration. If the lines *a b* and *b c*, Fig. 7, are supposed to represent the legs of a pair of compasses, the arc *d* will represent the path through which a point on the leg *a b* will have to move in order that this point may coincide with another point on the leg *b c*, at the same distance from *b*. The length of the arc *d*, in degrees, etc., will, therefore, serve to indicate the size of the angle.

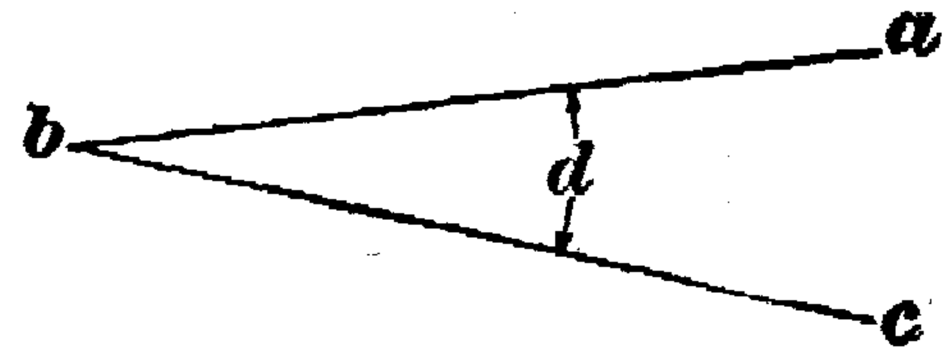

FIG. 7

In referring to an angle, three letters are commonly required, one letter at a point on each leg and one at the vertex. In naming the angle, the letter at the vertex is always placed between the other two. Thus, the angle, Fig. 7, is referred to as the angle *a b c*. If the angle stands by itself, as in this case, it may be referred to simply by the letter at the vertex, as angle *b*. Angles are sometimes referred to by means of a short arc and a letter, as in Fig. 7, where *d* indicates the angle.

13. Classification of Angles.—When two lines cross, as in Fig. 8 (*a*) and (*b*), they form four angles *a o c, c o b, b o d,* and *d o a*. Angles that have the same vertex and a common side between them are **adjacent angles,** as *a o c* and *c o b*, in which *o* is the vertex of each and *o c* is common to both angles.

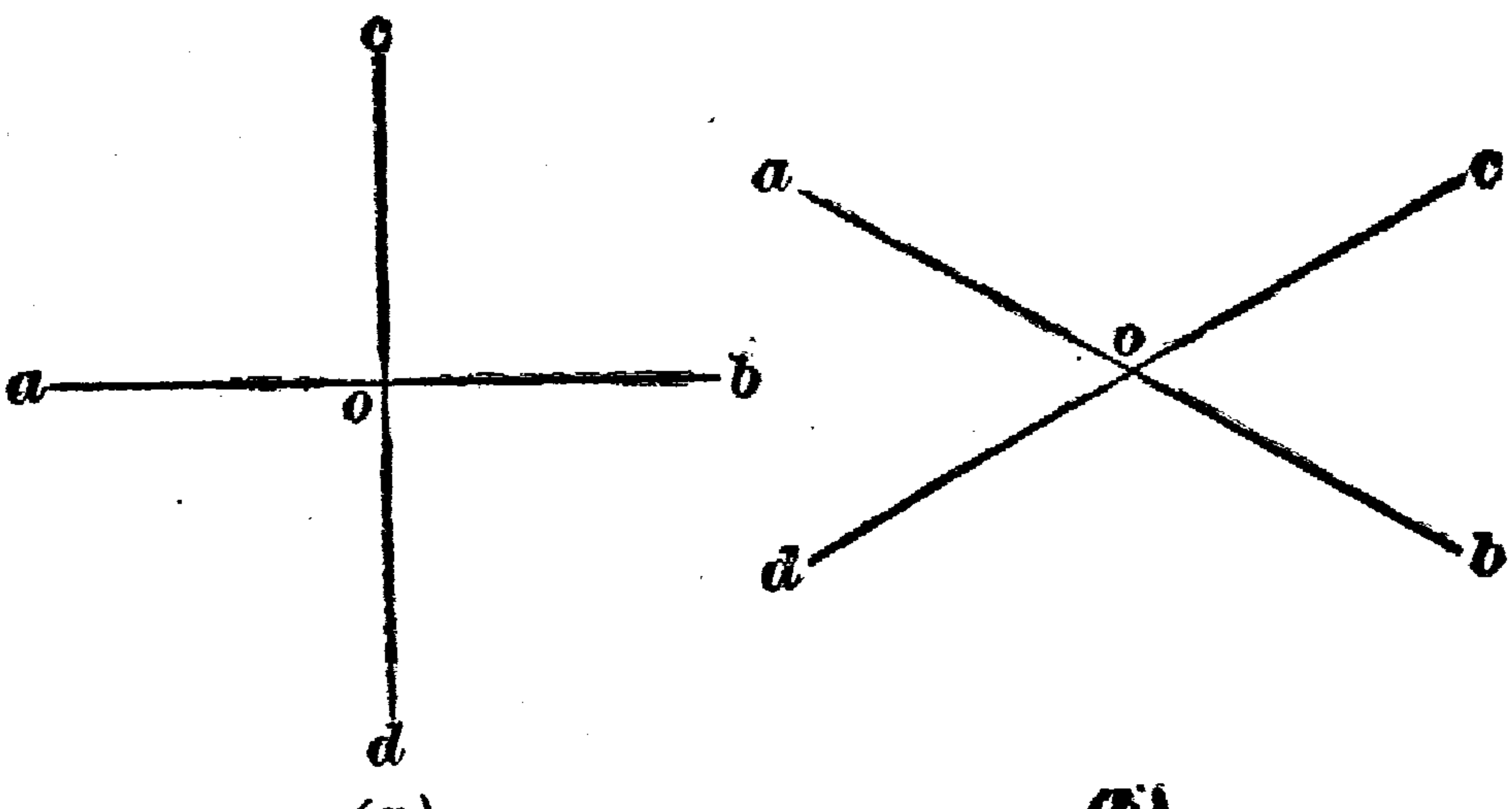

FIG. 8

When two lines intersect, the angles formed on opposite sides of the vertex are **opposite angles.** For instance, the angles made by a pair of scissors blades and by the handles, respectively, are opposite angles. In Fig. 8, *a o c* and *b o d* are opposite angles; so are *a o d* and *b o c*.

14. When one straight line meets another so that the adjacent angles formed are equal, as *a o c* and *c o b*, Fig. 8 (*a*), the angles are called **right angles,** and the lines are said to be **at right angles,** or **normal,** or perpendicular, to each other. When a line is perpendicular to another line, as in Fig. 8 (*a*), the adjacent angles must be right angles.

The sum of all the angles that are on one side of a straight line and have a common vertex is always equal to two right angles. In Fig. 8 (*a*) and (*b*) the sum of the angles *a o c* and *c o b* is equal to two right angles. If additional straight lines are drawn through the vertex *o* the number of angles will increase, but the sum of the angles on the *same* side of the line *a b* will remain equal to two right angles.

If there are two right angles on one side of a straight line, as in the case of the line *a b,* Fig. 8 (*a*), there must also be two right angles on the other side. Hence, the sum of all angles about a common vertex is equal to four right angles.

15. An angle greater than a right angle, as the angle *a o c,* Fig. 8 (*b*), is an **obtuse angle,** and an angle less than a right angle, as the angle *c o b,* is an **acute angle.**

16. Complements and Supplements.—An angle is said to be the complement of another, when the sum of the two angles is *one* right angle. In Fig. 9, if *b o* is at right angles to *a d,* the angle *b o c* is the complement of the angle *c o d,* because the sum of the two angles is a right angle, *b o d.* Two angles that together make one right angle are said to be **complementary.** Thus, *b o c* and *c o d* are complementary angles.

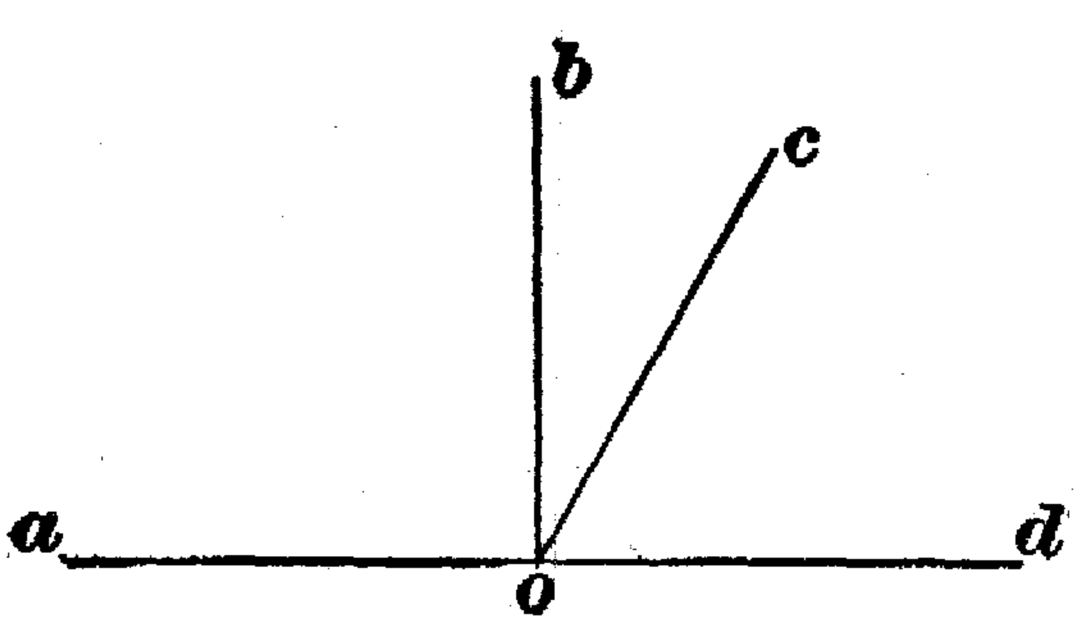

Fig. 9

When the sum of two angles is equal to *two* right angles, the angles are said to be **supplementary,** and each is the **supplement** of the other. In Fig. 9, the angle *c o d* is the supplement of the angle *c o a,* and *c o a* is the supplement of *c o d.* Also, the angles *a o b* and *b o d* are supplementary angles.

17. Angular Measure.—If the length of the arc *d,* Fig. 7, is to serve as a measure of the size of an angle, it is necessary to select a suitable unit. For this purpose the circumference of a circle is divided into 360 equal parts called **degrees,** and each degree is divided into 60 equal parts known as **minutes.** A minute is divided into 60 equal parts called **seconds,** which are used where greater accuracy is required than can be expressed in degrees and minutes.

If two lines are drawn at right angles to each other, as in Fig. 8 (*a*), the point of intersection, *o*, may be used as the center of a circle. This circle will then be divided by the intersecting lines into four equal parts, known as **quadrants,** each representing an angle of $\frac{360}{4}=90$ degrees, usually written 90°, as shown in Fig. 10. One-half of a right angle is $\frac{90}{2}=45°$, one-fourth is $22\frac{1}{2}°$, one-third is 30°, two-thirds is 60°, etc., as indicated.

If the radius of the circle, drawn with the vertex of the angle as a center, is increased in length, the circumference of the circle will also become greater, and, consequently, also the length of each division. But the angle, or amount of opening, included between two intersecting lines will not be affected. Thus, the angle marked 90° in Fig. 10 will include $\frac{1}{4}$ of 360°, as before.

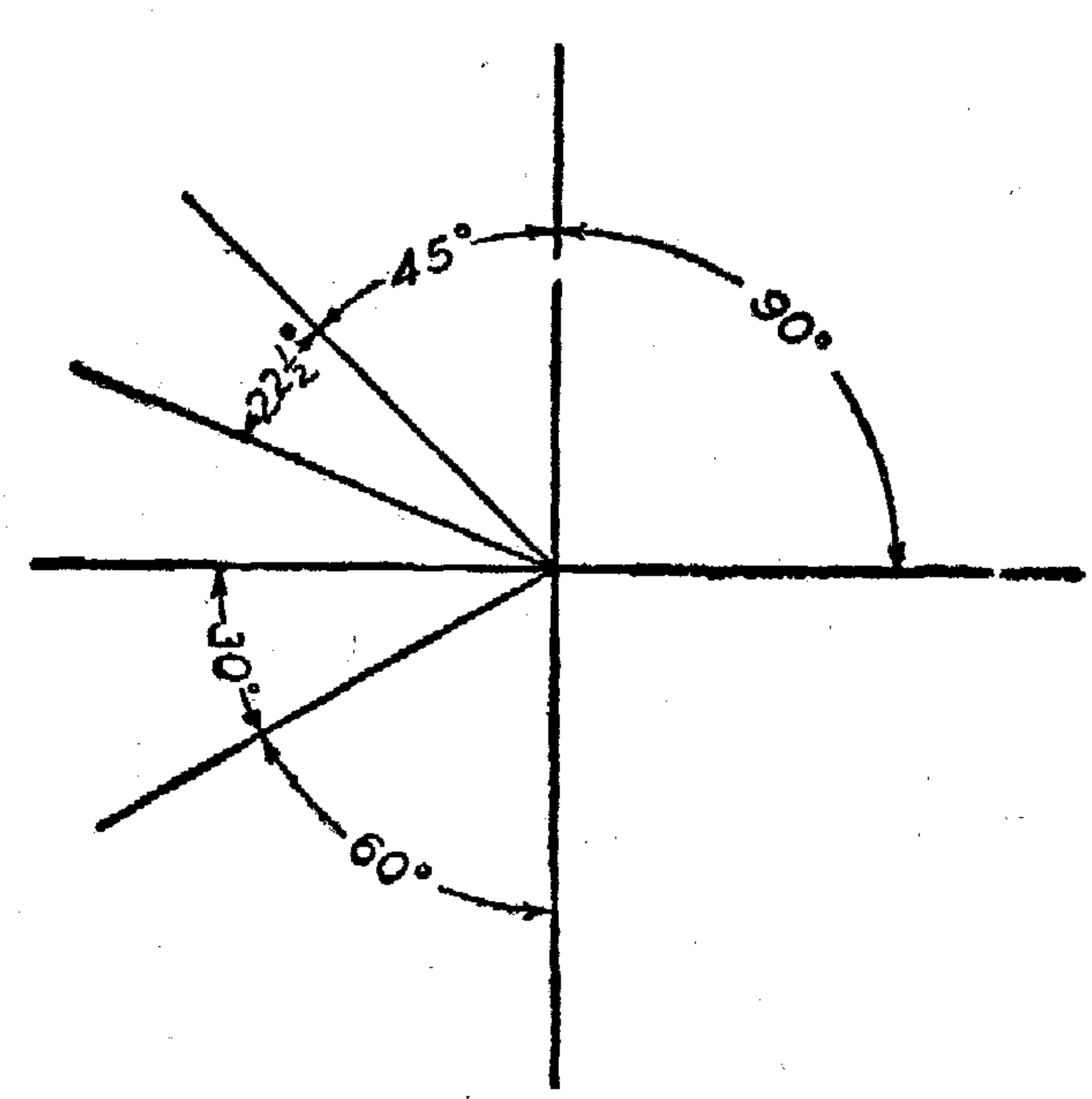

FIG. 10

This system of measuring angles is called **angular measure;** its units and abbreviations are given in the following table. The signs for minutes and seconds are ′ and ″, the same as for feet and inches; but as they usually occur in connection with the sign for degrees, there is little danger of their being misread for feet and inches. The expression 27° 13′ 45″ is read *27 degrees 13 minutes 45 seconds.*

Usually the mechanic will not have to deal with smaller divisions of a degree than minutes; that is, the values of the angles he may have to use in calculations will ordinarily be given in degrees and minutes, and not in degrees, minutes, and seconds.

ANGULAR MEASURE

60 seconds (″)	=1 minute		′
60 minutes	=1 degree		°
360 degrees	=1 circle		cir.

18. Calculations in Angular Measure.—In some calculations it may be necessary to reduce degrees and minutes to degrees and a fraction of a degree; that is, to reduce a denominate number to a higher denomination, as explained in *Weights and Measures*. To do so, the number of minutes is written as the numerator of a fraction with 60 as the denominator, and added to the whole number of degrees. For example, 12° 45′ is equal to $12\frac{45}{60}$ degrees, or $12\frac{3}{4}$ degrees, which may be expressed decimally as 12.75°. If the expression contains both minutes and seconds, the minutes and seconds are reduced to seconds and divided by 3,600, the number of seconds in one degree, and the fraction is added to the whole number of degrees. For example, if 39° 16′ 15″ is to be reduced to degrees, the expression 16′ 15″, reduced to seconds, becomes $(16\times60)+15=975$ seconds. Dividing this product by 3,600, the fraction is $\frac{975}{3600}=\frac{13}{48}=.271$. Therefore, 39° 16′ 15″ may be written $39\frac{13}{48}$ degrees, or 39.271°.

19. Angles may be added and subtracted in the same way as other compound numbers. For example, if the angles of 45° and $22\frac{1}{2}$°, Fig. 10, are to be added, their sum is $45°+22\frac{1}{2}° =67\frac{1}{2}°$.

The following examples show how to proceed in calculations dealing with angular measure.

EXAMPLE 1.—Find the sum of 12° 34′, 7° 48′, and 36° 11′.

SOLUTION.—The compound numbers are arranged as shown, with like units in the same column. The sum of the right-hand column is 93′, which is equal to 1° 33′. The number 33 is written under the minutes, the 1° is carried over and added to the degree column, giving a total of 56°. The sum is then 56° 33′.

12°	34′	
7	48	
36	11	
56°	33′	Ans.

EXAMPLE 2.—Subtract 12° 25′ from 45° 40′.

SOLUTION.—The smaller value is placed under the larger and subtracted in the usual manner.

45°	40′	
12	25	
33°	15′	Ans.

EXAMPLE 3.—From 39° 4′ subtract 27° 56′.

SOLUTION.—As 56′ cannot be taken from 4′, 1°, or 60′, is taken from the 39°, leaving 38°, and added to the 4′, making 64′. The minuend will then assume the form as shown below, and the subtraction may now be carried out in the usual manner.

$$\begin{array}{rr} 39° & 4' \\ 27 & 56 \\ \hline \end{array}$$

$$\begin{array}{rr} 38° & 64' \\ 27 & 56 \\ \hline 11° & 8' \end{array} \text{ Ans.}$$

The minuend is not usually rewritten as shown here, but the process of taking 1° from 39° and adding its equivalent, or 60′, to the minutes is done mentally.

20. Means for Measuring Angles.—Angles are commonly measured by the use of a **protractor,** a form of which is shown in Fig. 11. It consists of a piece of celluloid or of metal of a semicircular shape and very thin. Along the curved edge are a number of divisions, the smallest representing ½°,

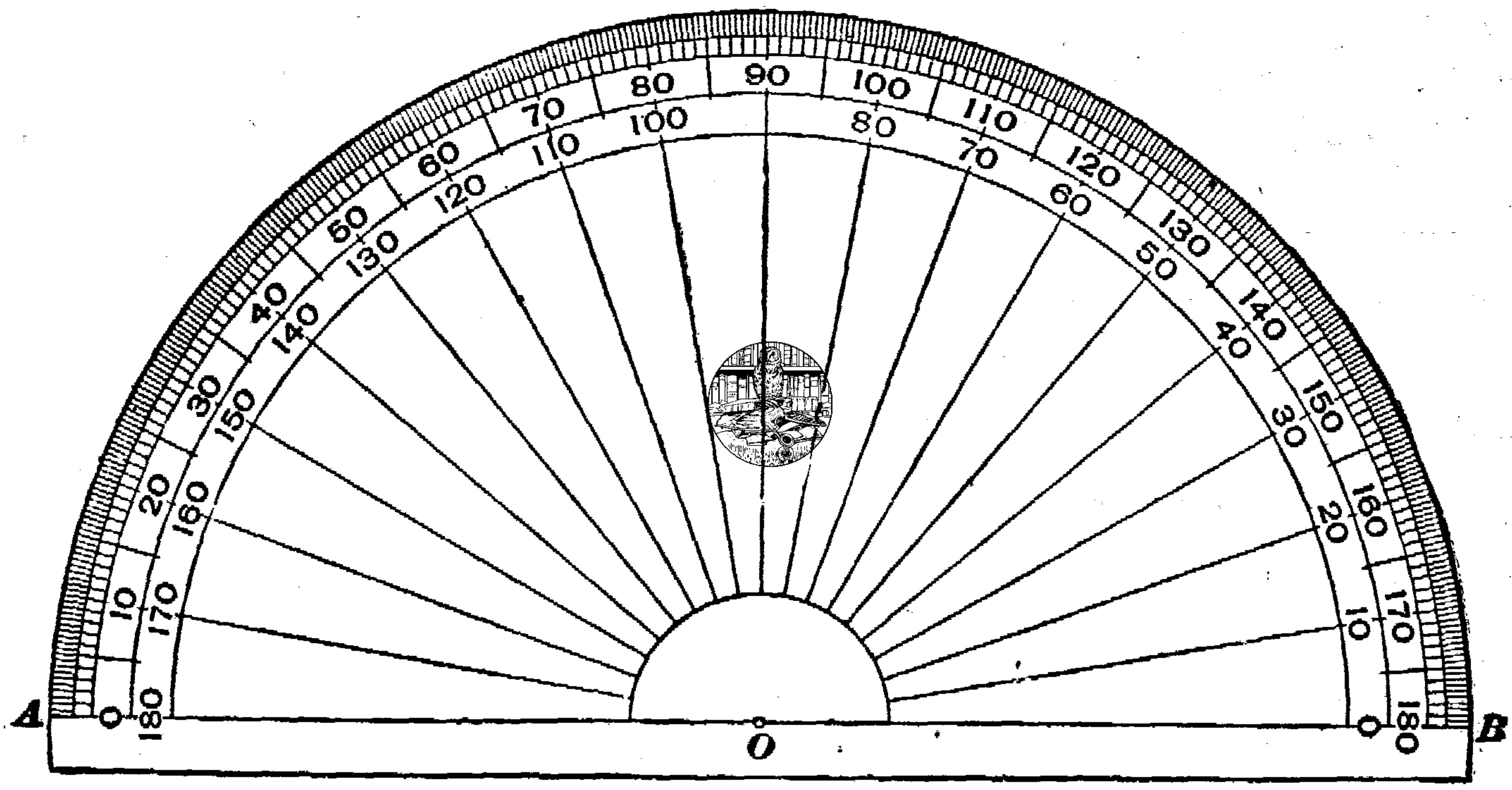

FIG. 11

or 30′, the next larger representing 1°, and the larger ones representing 5° and 10°, respectively. To use the protractor, it is laid flat on the angle to be measured, with the point *O* directly on the vertex of the angle and the line *A B* directly over one side of the angle. The point where the other side of the angle crosses the scale shows the size of the angle.

21. Laying Off an Angle.—If a line is given and it is required to draw another line that inclines toward it at a given angle, the required line may be drawn in the following manner: Let *C F*, Fig. 12, be the given line and *C* the point from which a line is to be drawn at an angle of 54° with the line *C F*. The protractor, Fig. 11, is laid on the line *C F* so that its center *O* comes directly over the vertex *C* of the angle and so that the line $A B$ coincides with the line $C F$, Fig. 12. Then 54° is counted off from the lower end B of the scale on the protractor. This means 5 large divisions, each of which represents 10°, and 4 small divisions, each of which represents 1°. Opposite the end of the 54° mark the point D is located with a sharp

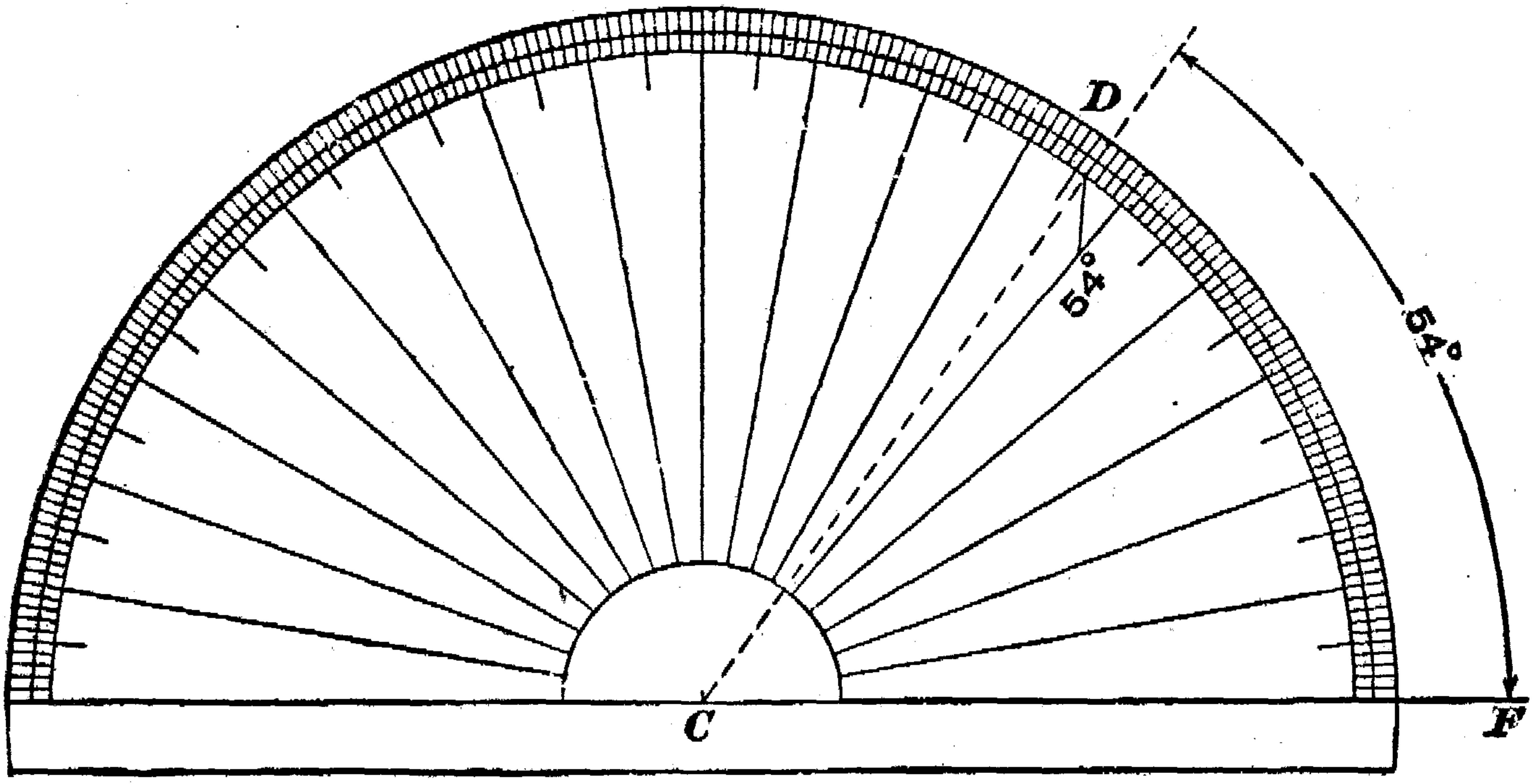

FIG. 12

pencil or a *scriber,* a pointed steel tool used for *scribing,* or scratching, lines on metal surfaces. Then the protractor is removed and a straight line is drawn or scribed so as to pass through C and D. This line $C D$ will make an angle of 54° with the line $C F$; that is, the angle $D C F$ will be 54°.

EXAMPLES FOR PRACTICE

1. Find the sum of 26° 18′ and 18° 42′. Ans. 45°

2. What is the difference between 88° 28′ and 42° 12′? Ans. 46° 16′

3. What is the complement of 36° 32′? Ans. 53° 28′

4. What is the supplement of 87° 29′? Ans. 92° 31′

5. Find the difference between the complement of 18° 30′ and the supplement of the same angle. Ans. 90°

6. Subtract 82° 48′ from 112° 23′. Ans. 29° 35′

7. How many seconds are there in 32° 14′ 6″? Ans. 116,046″

8. How many degrees, minutes, and seconds do 38,582 seconds amount to? Ans. 10° 43′ 2″

9. In a pulley with five arms, what part of a right angle is included between the center lines of any two adjacent arms?

Ans. $\frac{4}{5}$ of a right angle

10. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, how many degrees are there in each angle? Ans. 30°

SURFACES

DEFINITIONS AND CLASSIFICATION

22. Plane Surface.—A **plane surface,** usually called a **plane,** is a surface upon which straight lines may be drawn in any direction. A practical example of a plane is the surface of a *surface plate,* a steel plate ground perfectly flat on its top surface. If a straightedge is laid on its surface, every point along the edge of the straightedge will touch the surface, no matter in what direction it is laid.

23. Plane Figures.—Any part of a plane surface bounded by any number of straight or curved lines or a combination of the two is known as a **plane figure.**

24. Polygons.—When a plane figure is bounded by straight lines only, it is called a **polygon.** The bounding lines are called **sides,** and the combined length of the sides is known as the **perimeter** of the polygon. Polygons include figures with three, four, or more sides, but it is customary to divide polygons into three classes, calling those with three sides *triangles,* those with four sides *quadrilaterals,* and those with more than four sides *polygons.*

TRIANGLES

25. Classification.—A **triangle** is a polygon with three sides. Triangles are named according to the relative lengths of their sides as *isosceles, equilateral,* and *scalene triangles,* and according to the nature of the angles as *right-angled* and *oblique triangles.*

26. When two sides of a triangle are of equal lengths, as in Fig. 13, it is known as an **isosceles** (i-sos′se-leez) **triangle.** This word *isosceles* means *equal legs.*

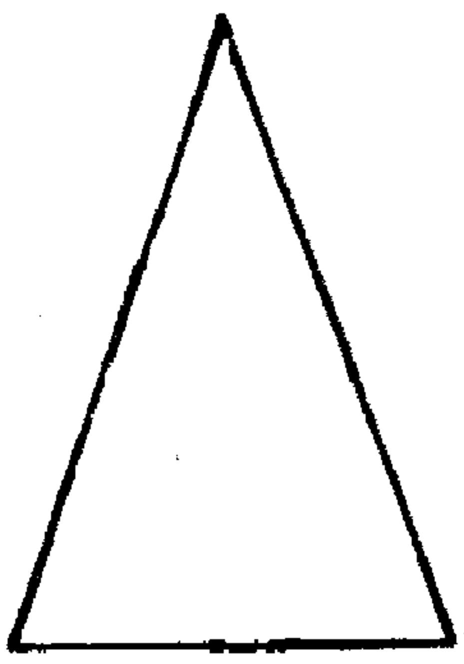

Fig. 13

27. When the three sides of a triangle are of equal lengths, as in Fig. 14, it is called an **equilateral triangle.**

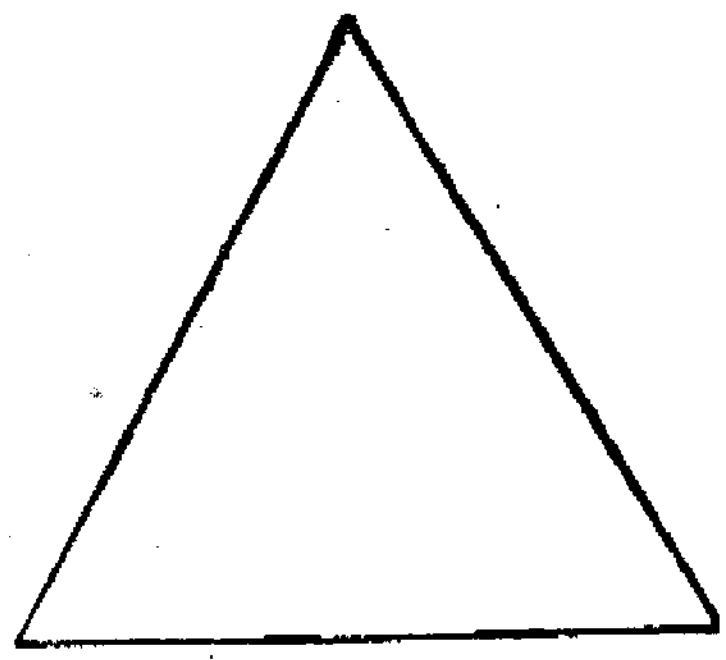

Fig. 14

28. A triangle in which all the sides are of different lengths, as Fig. 15, is called a **scalene triangle.**

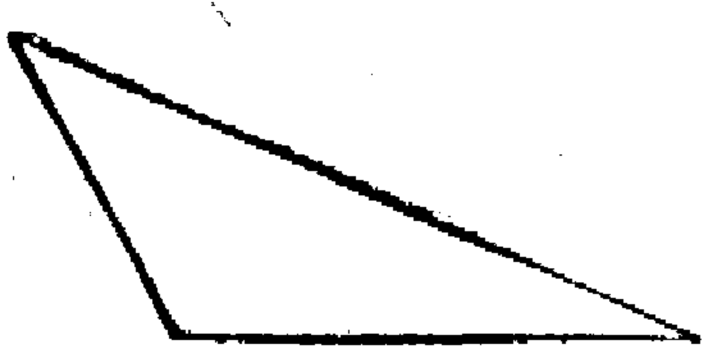

Fig. 15

29. If one of the angles in a triangle is a right angle, the triangle is a **right-angled triangle.** The side opposite the right angle is called the **hypotenuse.** In Fig. 16 the side *A B* is the hypotenuse because it is opposite the right angle *C*. For brevity, a right-angled triangle is generally termed a **right triangle.**

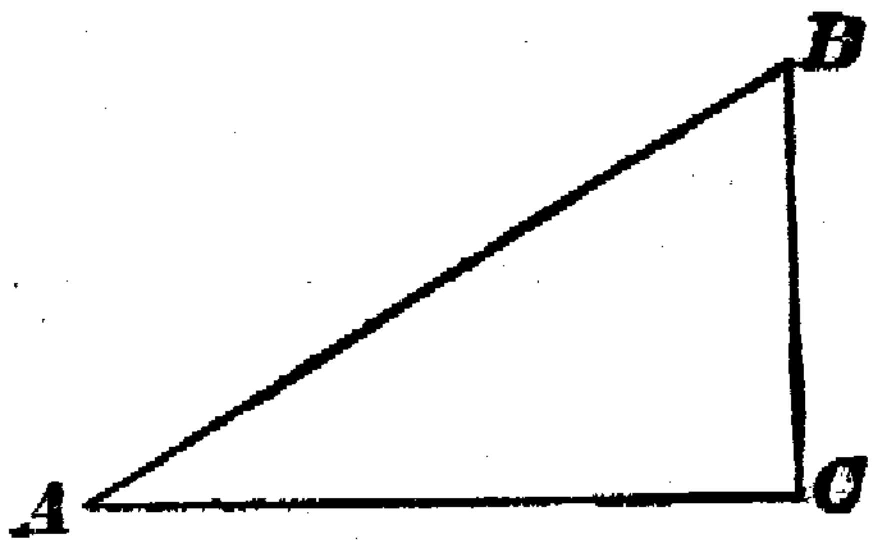

Fig. 16

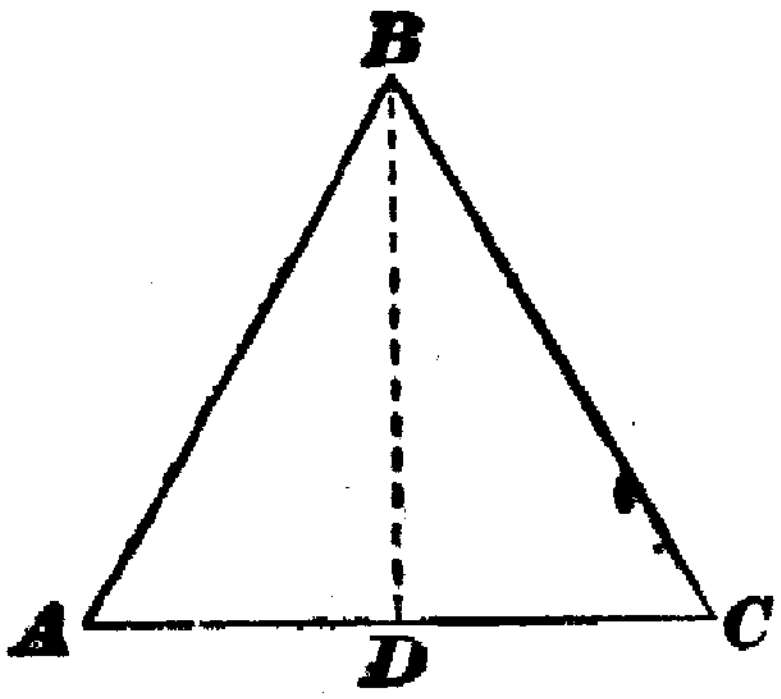

Fig. 17

30. Base and Altitude.—That side of a triangle on which it is supposed to stand is known as the **base;** any side may be considered as the base. In Figs. 16, 17, and 18, *A C* is shown as the base.

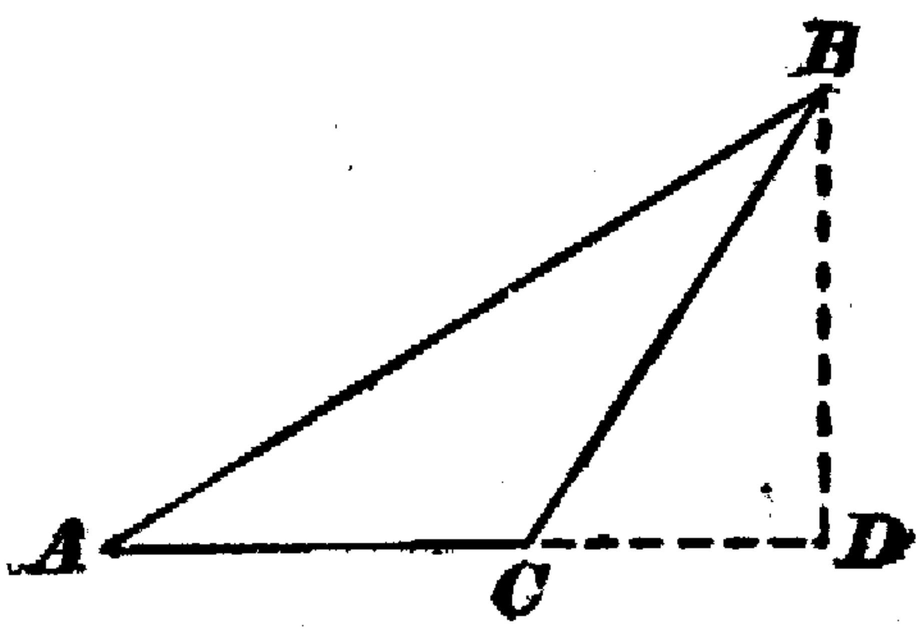

Fig. 18

The **altitude, or height,** of any triangle is represented by a line drawn from the vertex of the angle opposite the base perpendicular to the base or to an extension of the base. Thus, in Figs. 17 and 18, *B D* is the altitude of the triangle *A B C*. In Fig. 18 the perpendicular falls outside the triangle, and the base *A C* is shown extended to meet the line *B D*.

31. Similar and Equal Triangles.—Two triangles are **similar** when the *angles* of one are equal to the angles of the other. If in the triangle *a b c*, Fig. 19, a line *d e* is drawn parallel to the side *b c*, the triangle *a d e* is similar to the triangle *a b c*, as their angles are equal, each to each.

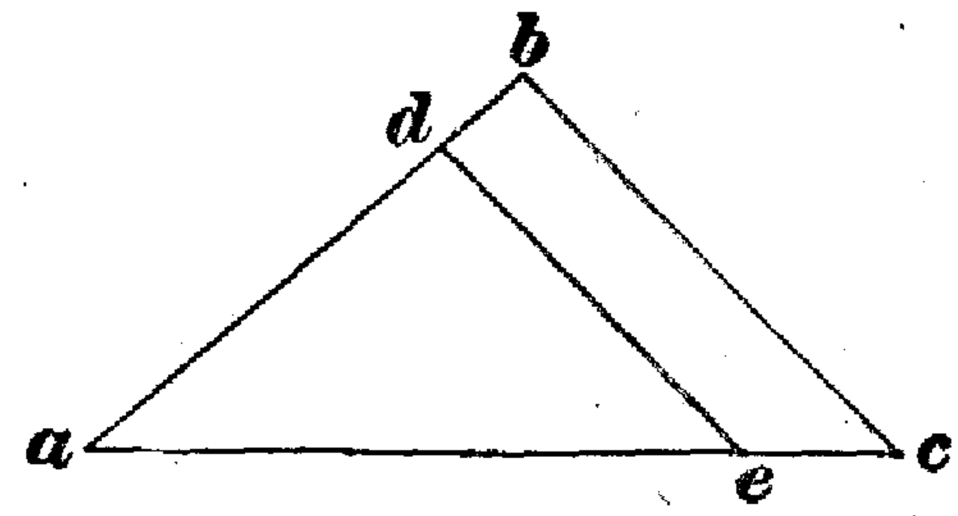

Fig. 19

Two triangles are **equal** when the *sides* of one are equal to the *sides* of the other. It is seen that triangles may be similar without being equal.

QUADRILATERALS

32. Parallelograms.—A **parallelogram** is a quadrilateral with opposite sides parallel and opposite angles equal. There are four kinds of parallelograms: the *rectangle*, the *square*, the *rhomboid*, and the *rhombus*.

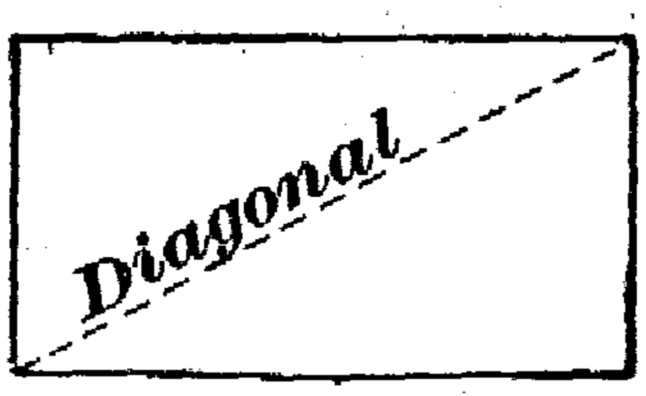

Fig. 20

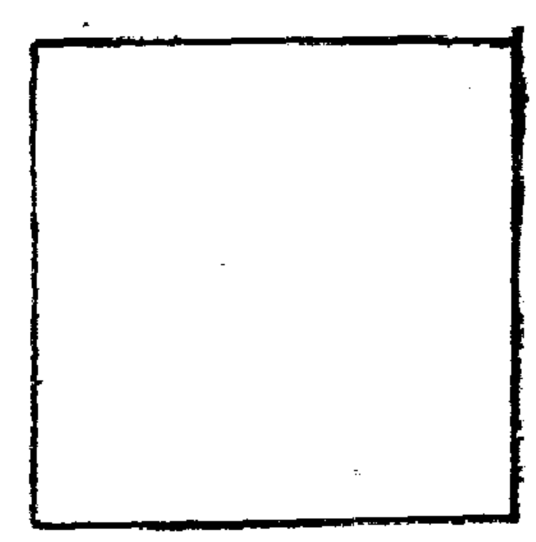
Fig. 21

33. A **rectangle**, Fig. 20, is a parallelogram with sides at right angles to one another. If the sides are equal, as in Fig. 21, the rectangle is known as a **square**.


Fig. 22

34. A **rhomboid**, Fig. 22, is a parallelogram in which there are no right angles and in which the length of one pair of opposite sides differs from that of the other pair.

A **rhombus**, Fig. 23, is a parallelogram without any right angles, and with sides of equal lengths.

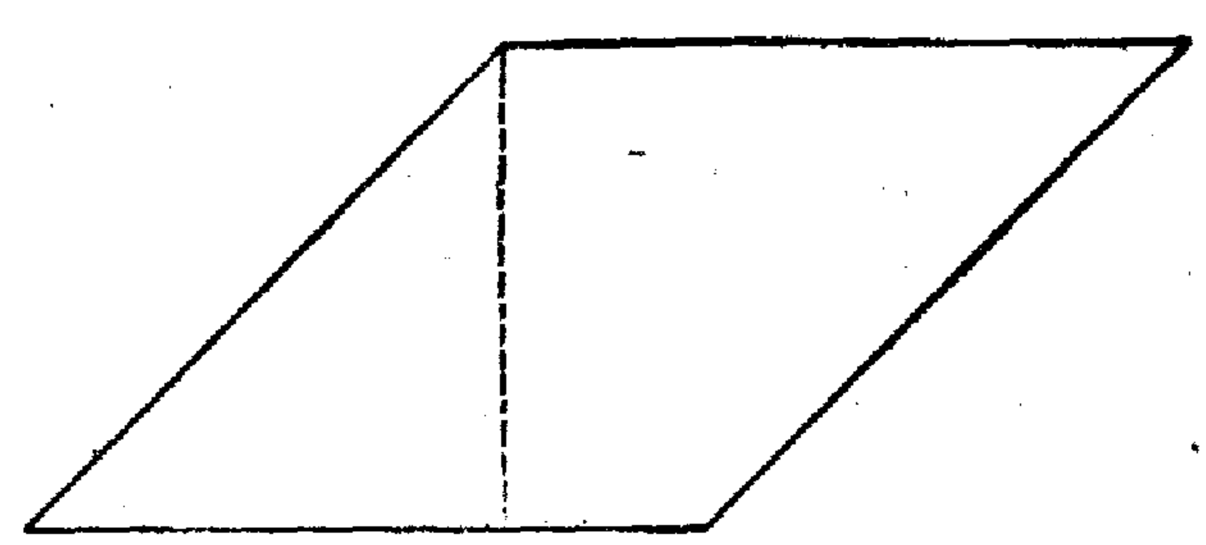
Fig. 23

35. Trapezoid and Trapezium.—A **trapezoid**, Fig. 24, is a quadrilateral which has only two of its sides parallel.

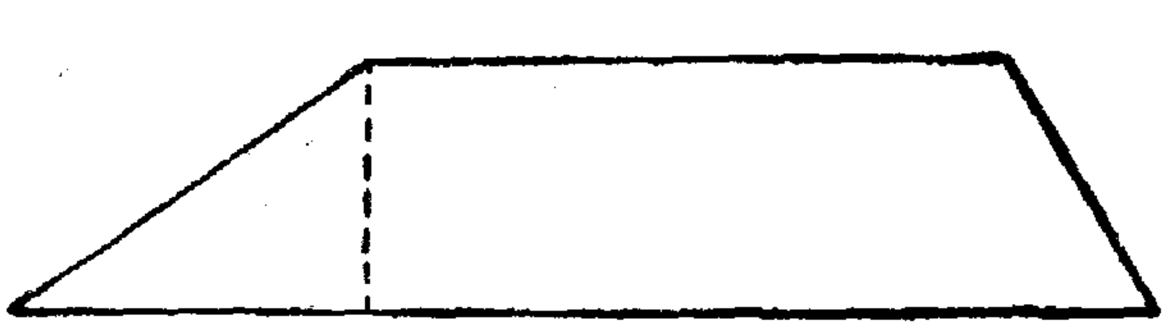
Fig. 24

A **trapezium**, Fig. 25, is a quadrilateral without any parallel sides.

36. Altitudes and Diagonals of Quadrilaterals. Any side of a quadrilateral may be considered as its **base**, but in a trapezoid one of the two parallel sides is usually considered as its base. The **altitude** of a parallelogram or of a trapezoid is the perpendicular distance between two parallel sides. Altitudes are indicated by the dotted lines in Figs. 22, 23, and 24.

37. A **diagonal** is a straight line joining two opposite corners of a quadrilateral, as indicated by the dotted line in Fig. 20. Each quadrilateral has two diagonals, and either diagonal divides the quadrilateral into two triangles. In a parallelogram, these triangles are equal; for example, either diagonal of the rectangle, Fig. 20, divides it into two equal triangles. It is important to remember that the two diagonals of any parallelogram **bisect** each other; that is, they divide each other into two equal parts.

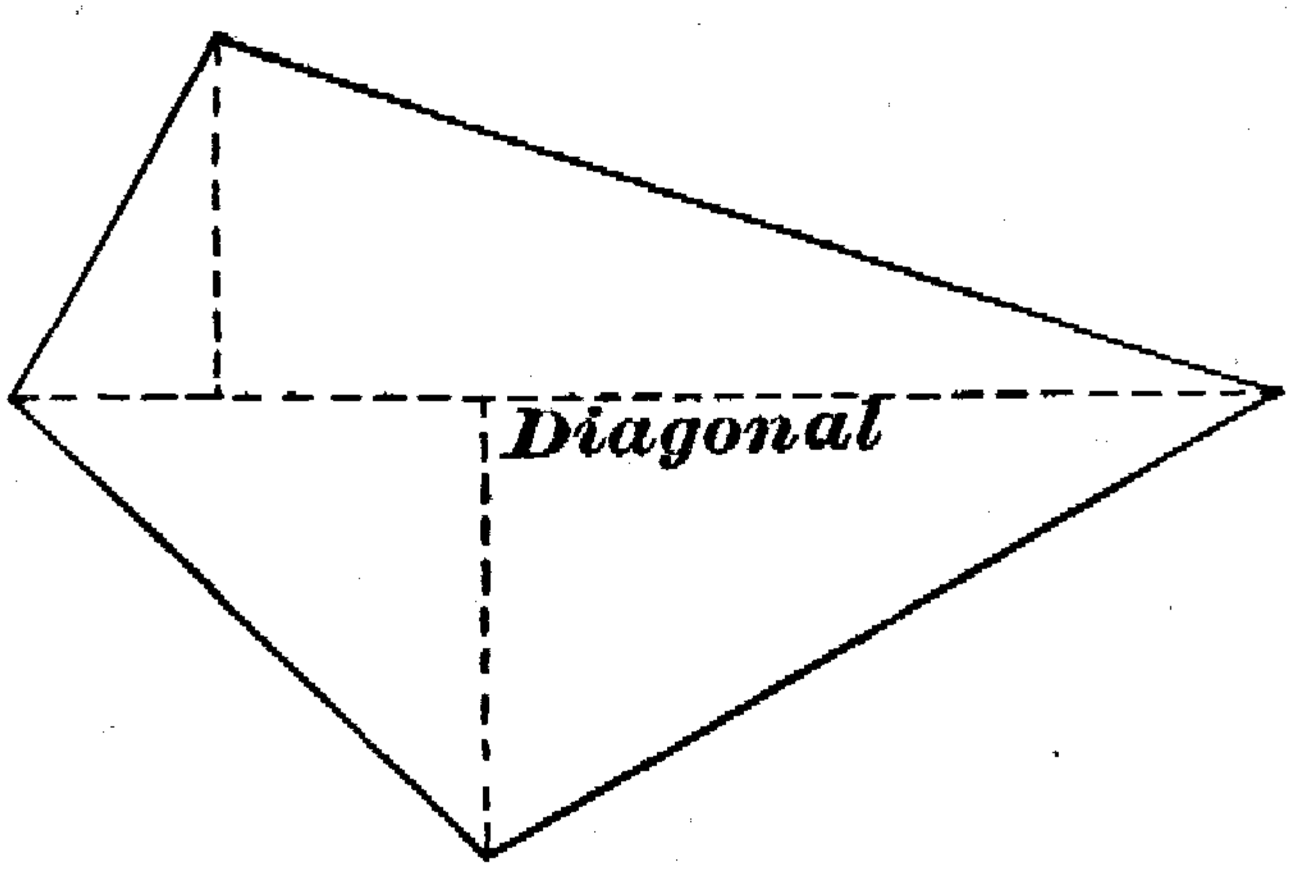

FIG. 25

REGULAR AND IRREGULAR POLYGONS

38. Regular Polygons. — Excluding, according to Art. **24,** any polygons with less than five sides, a **regular polygon** may be defined as one with five or more sides of equal lengths and with equal angles. Fig. 26 shows some common regular polygons with more than four sides. At (*a*) is shown the *pentagon,* or five-sided figure; at (*b*) the *hexagon,* or six-sided figure; at (*c*) the *heptagon,* or seven-sided figure; and at (*d*) the *octagon,* or eight-sided figure.

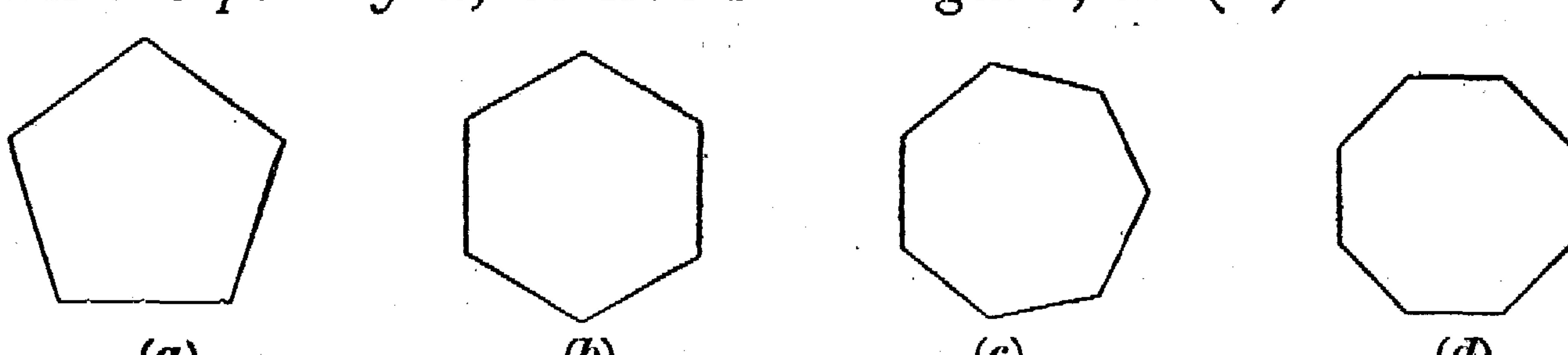

FIG. 26

It is true that equilateral triangles and squares are regular polygons of three and four sides, respectively, but they are not commonly called polygons.

39. Constructing Regular Polygons.—If perpendiculars are erected to all the sides of a regular polygon at