WEIGHTS AND MEASURES

Serial 1978 Edition 1

DENOMINATE NUMBERS

ENGLISH MEASURES

DEFINITIONS

1. Varieties of Measures.—A **measure** is a standard unit, established by law or custom, by means of which a quantity of any kind may be measured. For example, the inch and the mile are *measures of length;* the pint and the gallon are *measures of capacity,* as used for liquids; the ounce and the ton are *measures of weight;* the second and the month are *measures of time,* and so on.

2. Denominate Numbers.—When a number is used in connection with measures, it becomes a **denominate number,** that is, a *named* number. Numbers are said to be of the *same denomination* when they are combined with similarly named units of measure, as 2 pounds and 7 pounds. A pound is a *unit of measure;* a foot, an hour are also units of measure, but of different kinds.

If a denominate number consists of units of measure of but one denomination, it is called a **simple denominate number** or a **simple number.** For example, 14 inches is a simple number, as it contains units of measure of but one denomination, namely, inches. The denominate numbers 16 cents, 10 hours, 6 gallons are all *simple* numbers, but they are not of the same *kind.*

If simple numbers of different denominations but of the same kind of measure are combined, the combination is called a compound denominate number, or a compound number. For example, 3 yards 2 feet 7 inches is a compound number, as it is a combination of the different denominations, yards, feet, and inches; these denominations are of the *same* kind, being all unit measures of length. Other examples of compound numbers are: 2 pounds 3 ounces 10 grains; 5 gallons 3 quarts 1 pint; 10 hours 14 minutes 32 seconds.

A unit measure that is larger than another unit measure is said to be of a **higher denomination;** if smaller than another unit measure, it is of a **lower denomination.** Thus, a *foot* is of a higher denomination than an *inch* and of a lower denomination than a *yard.*

3. Systems of Measures.—In modern practice two systems of measures are employed: the *English system* and the *metric system.* The **English system** is in general use in the United States, Great Britain, and Canada. The **metric system** is used on the European continent and to some extent in the United States, as, for instance, in chemistry and pharmacy. This system is a decimal system; that is, the values of the different units of the same kind increase or decrease by tens, 10 units of each denomination making 1 unit of the next higher denomination.

ABBREVIATIONS

4. Abbreviations of Units.—In writing denominate numbers, it is convenient to use *abbreviations* instead of writing out the name of the unit in full. For example, the names *inch* and *inches* may be abbreviated to *in.;* thus, write 5 *in.* instead of 5 *inches.* Similarly, 8 *ft.* means 8 *feet.* Sometimes, the marks (′) and (″) are used for feet and inches, respectively, in particular on drawings; thus 6′ 2″ means 6 feet 2 inches. The abbreviations commonly used for the various units are given in the following tables, each table showing the relation between various units of measure of the same kind. These tables should be memorized, including the abbreviations. The practice of writing the tables from memory will be found helpful.

MEASURES OF EXTENSION

LINEAR MEASURE

5. Definitions.—Measures of extension are used to measure lengths of lines, areas of surfaces, and contents of solids. When speaking of lines it is to be understood that a *mathematical line* has no breadth or thickness and merely indicates extension in one direction; that is, *length*. A *straight line* is frequently defined as the shortest distance between two points. The lengths of lines are expressed in **linear measures.**

TABLE OF LINEAR MEASURE

		Abbreviations
12 inches (in.)	=1 foot	ft.
3 feet	=1 yard	yd.
5½ yards / 16½ feet	=1 rod	rd.
320 rods / 5,280 feet	=1 statute mile	mi.
6,080 feet	=1 nautical mile	naut. mi.
3 nautical miles	=1 nautical league	naut. l.

NOTE.—The statute mile is used for measuring distances on land, lakes, and rivers. The nautical mile is used for measuring distances on the ocean.

EXAMPLE 1.—A piece of shafting is 7 feet long. What is its length in inches?

SOLUTION.—According to the table, 1 ft.=12 in.; therefore, 7 ft. must be equal to

$$7 \times 12 = 84 \text{ in. Ans.}$$

EXAMPLE 2.—The width of a door is 48 inches. What is the width in feet?

SOLUTION.—According to the table, 12 in.=1 ft.; therefore, the number of feet in 48 in. is

$$48 \div 12 = 4 \text{ ft. Ans.}$$

EXAMPLE 3.—How many feet are there in 3 statute miles?

SOLUTION.—According to the table, 5,280 ft.=1 mi.; therefore, 3 mi. must be equal to

$$3 \times 5{,}280 = 15{,}840 \text{ ft. Ans.}$$

LAND MEASURE

6. Surveyor's Linear Measure.—Surveyor's measure, also known as **land measure,** is used for measuring land, as in locating and laying out railways, roads, and tracts or building plots.

TABLE OF SURVEYOR'S LINEAR MEASURE

		Abbreviations
7.92 inches	=1 link	li.
100 links / 66 feet	=1 chain	ch.
80 chains	=1 statute mile	mi.

The surveyor's chain (also called Gunter's chain) of 100 links, each 7.92 inches long, has been used very extensively in the past and is still common, but its use is decreasing. Chains or steel tapes 50 or 100 feet in length, graduated in feet and decimals of a foot and sometimes in feet and inches, are being employed to a large extent at the present time.

Note.—In Mexico, and in those parts of the United States that belonged to Mexico previous to 1845, the old Mexican measures of length are sometimes used, and referred to in early surveys. The principal units are:

1 vara=2.75 feet=33 inches, English measure
5,000 vara=1 league=2.604 miles, English measure

Example 1.—How many inches are there in 50 links?

Solution.—According to the table 1 li.=7.92 in.; therefore, 50 links must be equal to

$$50\times 7.92=396 \text{ in. Ans.}$$

Example 2.—A line set out by a surveyor is found to measure 40 chains in length. What is the equivalent length in feet?

Solution.—According to the table, 1 ch.=66 ft.; therefore, 40 chains must be equal to

$$40\times 66=2{,}640 \text{ ft. Ans.}$$

Example 3.—A boundary line between two estates is found to measure 440 chains in length. What is the equivalent length in statute miles?

Solution.—According to the table, 1 mi.=80 ch. The problem consists of finding how many times 80 ch. is contained in 440 ch.; or,

$$440\div 80=5\tfrac{1}{2} \text{ mi. Ans.}$$

SQUARE MEASURE

7. Definitions.—The term **area** means the extent of a surface within its boundary lines. For example, the area of a floor is the extent of the visible surface limited by the four surrounding walls. As area refers to extent of surface, the unit of measurement must also be a surface. The **unit of area** is a *square,* which is a surface of the form shown in Fig. 1.

By the term **square** is meant a four-sided figure in which the sides are of equal length and at *right angles* to one another; that is, any two adjoining sides meet each other squarely. A **square inch** is a square that is 1 inch long on each side. A **square foot** is a square that measures 1 foot on each side, as

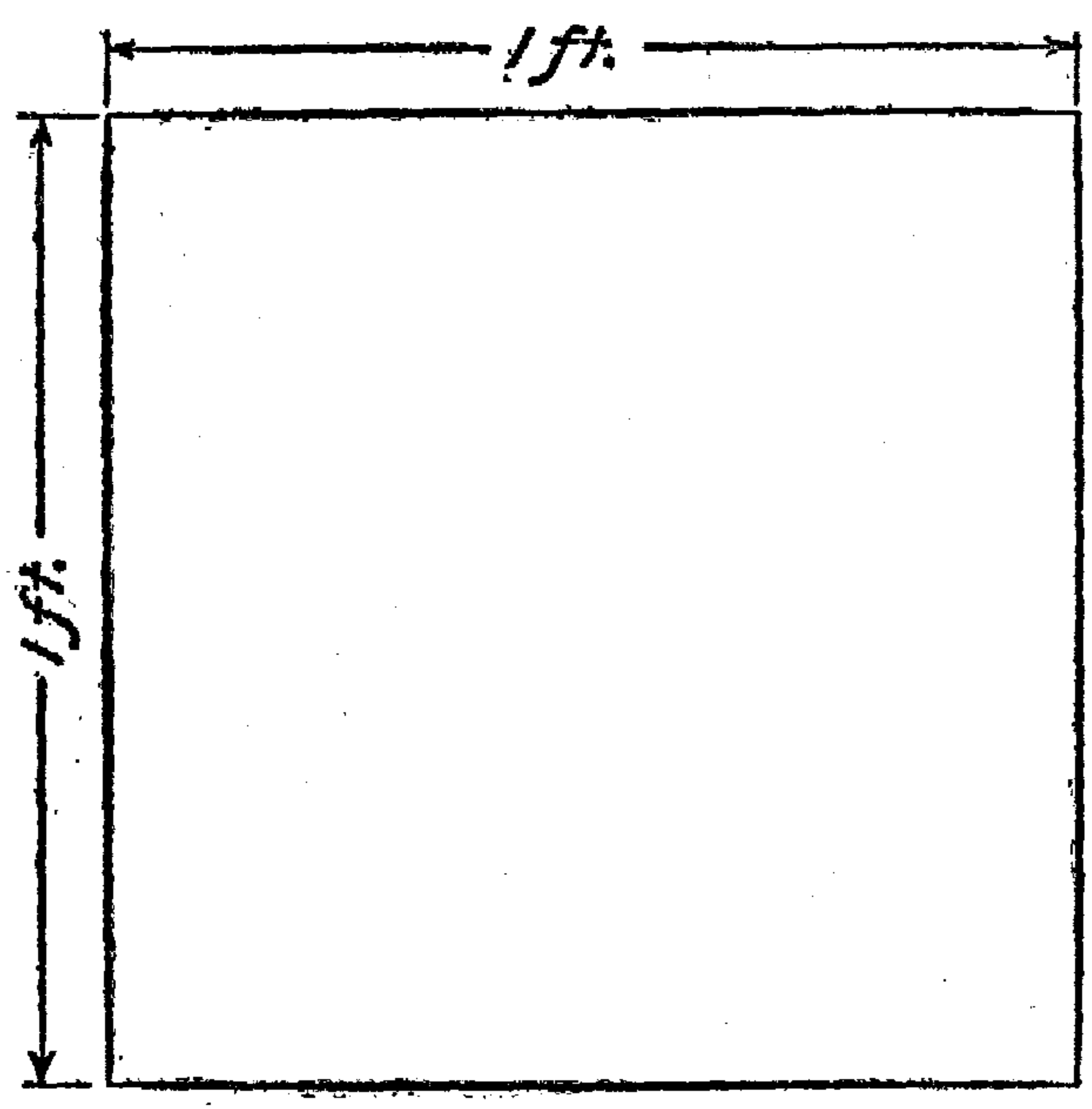

FIG. 1

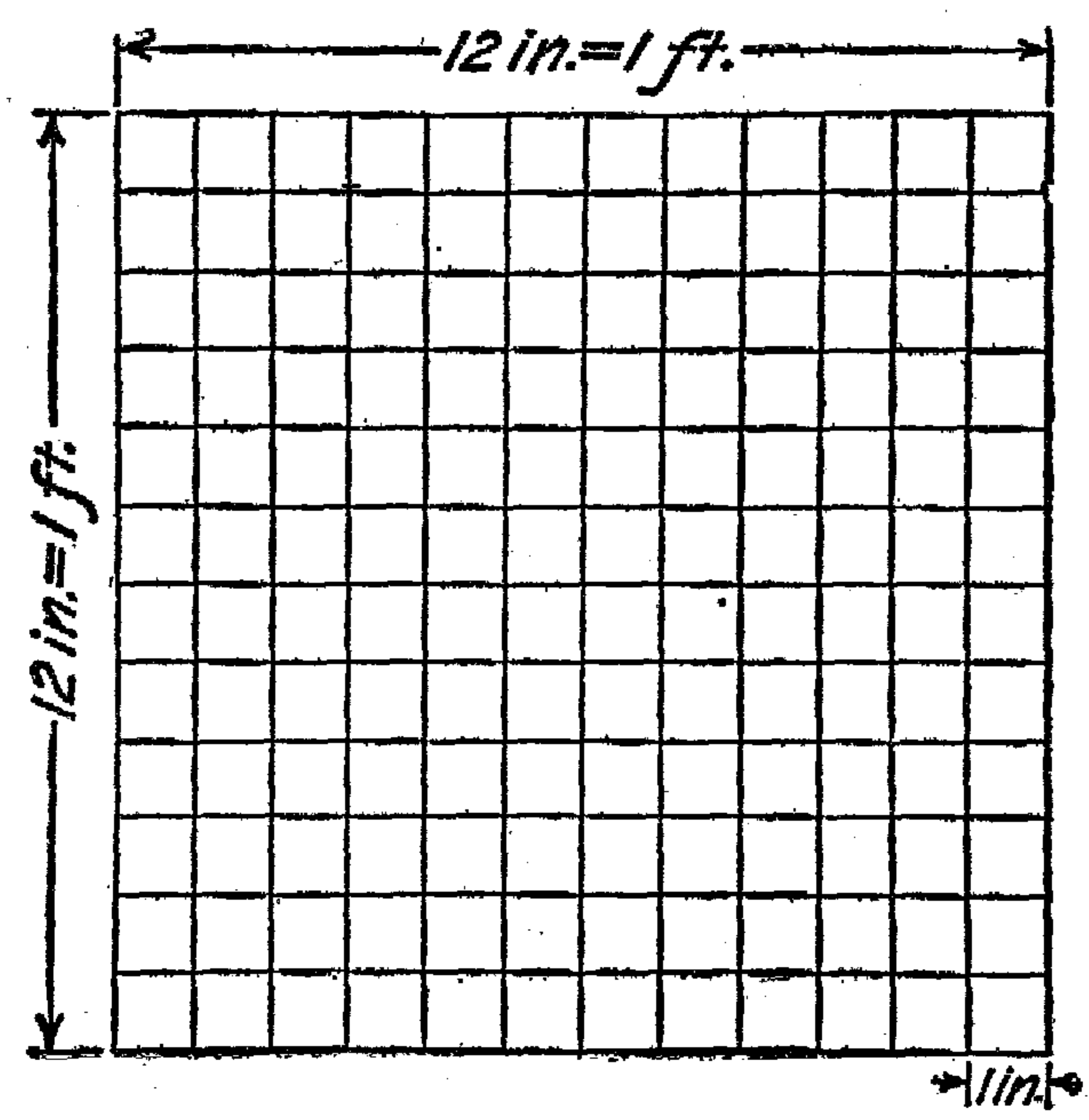

FIG. 2

in Fig. 1. This may be made to represent square inches by dividing each side into twelve equal parts, 1 foot being equal to 12 inches, and drawing lines across the square from the points of division on one side to those on the opposite side. The square will then appear as in Fig. 2, divided into a number of small squares. As each side of the square was divided into twelve equal parts, each part is $\frac{1}{12}$ foot long, or $\frac{1}{12}\times 12 = 1$ inch long, and each little square therefore measures 1 inch on each side; that is, each little square is 1 square inch. Now, if the total number of little squares is counted, it will be found to be 144. In other words, there are 144 square inches in 1 square foot.

8. Application of Square Measure.—A square inch, a square foot, etc. belong to the units of **square measure,** which is employed for measuring the extent of areas, such as floors, building lots, estates, etc. Square measure is also used to determine the surface areas of objects, such as engine cylinders, condensers, pipes, etc. Small areas are measured in square inches or square feet, and larger areas in square yards, square rods, acres, and square miles. The accompanying table gives the principal units of square measure.

TABLE OF SQUARE MEASURE

			Abbreviations
144	square inches (sq. in.)	=1 square foot	sq. ft.
9	square feet	=1 square yard	sq. yd.
30¼	square yards	=1 square rod	sq. rd.
160	square rods	=1 acre	A.
640	acres	=1 square mile	sq. mi.

9. Finding Areas of Squares and Rectangles. Instead of finding the area of a square by counting the number of small squares contained by it, as in Art. 7, it is possible to find the area more easily by simply multiplying the length of the square by its width. Thus, referring to Fig. 2, there are 12 rows of the little squares and 12 squares in each row or 12×12=144 square inches, which is the same result as obtained by counting. When this method of multiplying the length by the width is employed it is important to note that the length and the width must be in the *same* units. In the case just mentioned, both the length and the width were in inches, and the area as a result is in square inches. With the length and the width in feet, the area will be 1 foot×1 foot=1 square foot.

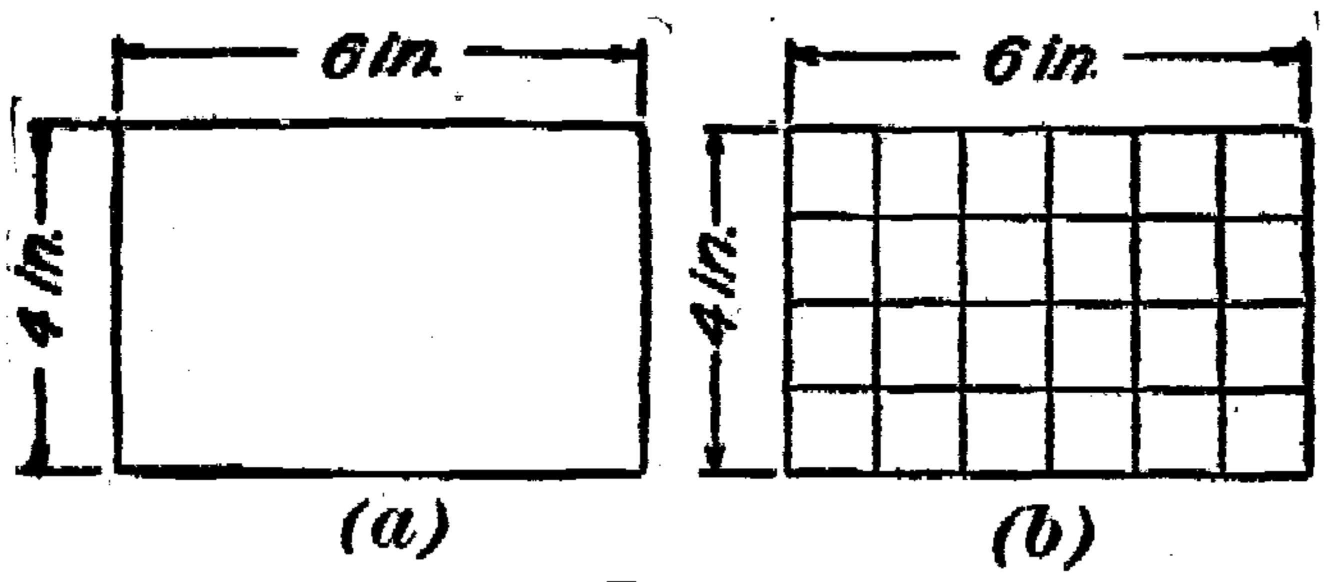

FIG. 3

Sometimes the area to be measured is a **rectangle;** that is, a figure similar to that shown in Fig. 3 (*a*). It differs from a square in so far that not *all* the sides are of equal length, but only those sides that are opposite each other. The area of a rectangle is found in the same way as that of a square; hence, the following rule applies to both figures:

Rule.—*To find the area of a square or a rectangle, multiply the length by the width, both being expressed in the same units.*

Thus, if the rectangle in Fig. 3 (*a*) is 6 inches long and 4 inches wide, its area is $6 \times 4 = 24$ square inches. This can be proved as shown in (*b*), by dividing each side into inches, drawing lines across the figure from the points of division, and counting the number of squares thus formed. Each of these squares is 1 inch on each side, or 1 square inch in area, and there are 24 in all, or 24 square inches.

10. Difference Between the Terms Square Feet and Feet Square.—Distinction must be made between expressions, such as "4 square feet" and "4 feet square," as they do not mean the same thing. When a square is referred to as being equal to *4 square feet,* reference is had to its *area.* It follows from the preceding rule that each of its sides must be of a length equal to 2 feet, in order that the area may be $2 \times 2 = 4$ square feet.

The expression "4 feet square" does not refer to the *area* of the square, but to *length of the sides,* and means that each side of the square is equal to 4 feet in length. The area of the latter square is $4 \times 4 = 16$ square feet.

11. Application of Rule for Finding Areas of Squares and Rectangles.—The application of the rule in Art. **9** is illustrated by means of the following examples:

Example 1.—How many square inches are there in a rectangle 28 inches long and 13 inches wide?

Solution.—According to the rule, the area is

$$28 \times 13 = 364 \text{ sq. in.} \quad \text{Ans.}$$

Example 2.—A sheet of copper is 4 feet long and 3 inches wide. What is the area of its surface?

Solution.—Both dimensions must be expressed in the same units; reducing 4 ft. to inches, the result is $12 \times 4 = 48$ in. Applying the rule, the area is

$$48 \times 3 = 144 \text{ sq. in., or } 1 \text{ sq. ft.} \quad \text{Ans.}$$

Example 3.—The base of an office cabinet is $6\frac{1}{4}$ feet long and $2\frac{1}{2}$ feet wide. What area of floor space does it cover?

SOLUTION.—According to the rule, the area is

$$6\tfrac{1}{4}\times 2\tfrac{1}{2}=\frac{25}{4}\times\frac{5}{2}=\frac{125}{8}=15\tfrac{5}{8}\text{ sq. ft.}\quad\text{Ans.}$$

12. Surveyor's Square Measure.—The square measure used by surveyors differs somewhat from that given in Art. **8.** The various units employed are included in the following table:

TABLE OF SURVEYOR'S SQUARE MEASURE

		Abbreviations
10,000 square links (sq. li.)	=1 square chain	sq. ch.
10 square chains	=1 acre	A.
640 acres	=1 square mile	sq. mi.
36 square miles	=1 township	Tp.

NOTE.—The acre contains 4,840 square yards or 43,560 square feet, and it is equal to the area of a square measuring 208.71 feet on each one of the sides.

EXAMPLE 1.—How many square links are there in 11 square chains?

SOLUTION.—According to the table, 1 sq. ch. is equal to 10,000 sq. li.; therefore, 11 sq. ch. must be equal to

$$11\times 10{,}000=110{,}000\text{ sq. li.}\quad\text{Ans.}$$

EXAMPLE 2.—A piece of land contains 85 square chains. What is the equivalent value in acres?

SOLUTION.—There are 10 sq. ch. in an acre; therefore, the number of acres contained in 85 sq. ch. must be

$$85\div 10=8.5\text{ A.}\quad\text{Ans.}$$

EXAMPLE 3.—How many acres are there in a township?

SOLUTION.—As one township contains 36 sq. mi. and each square mile contains 640 A., it follows that the number of acres contained in one township must be

$$36\times 640=23{,}040\text{ A.}\quad\text{Ans.}$$

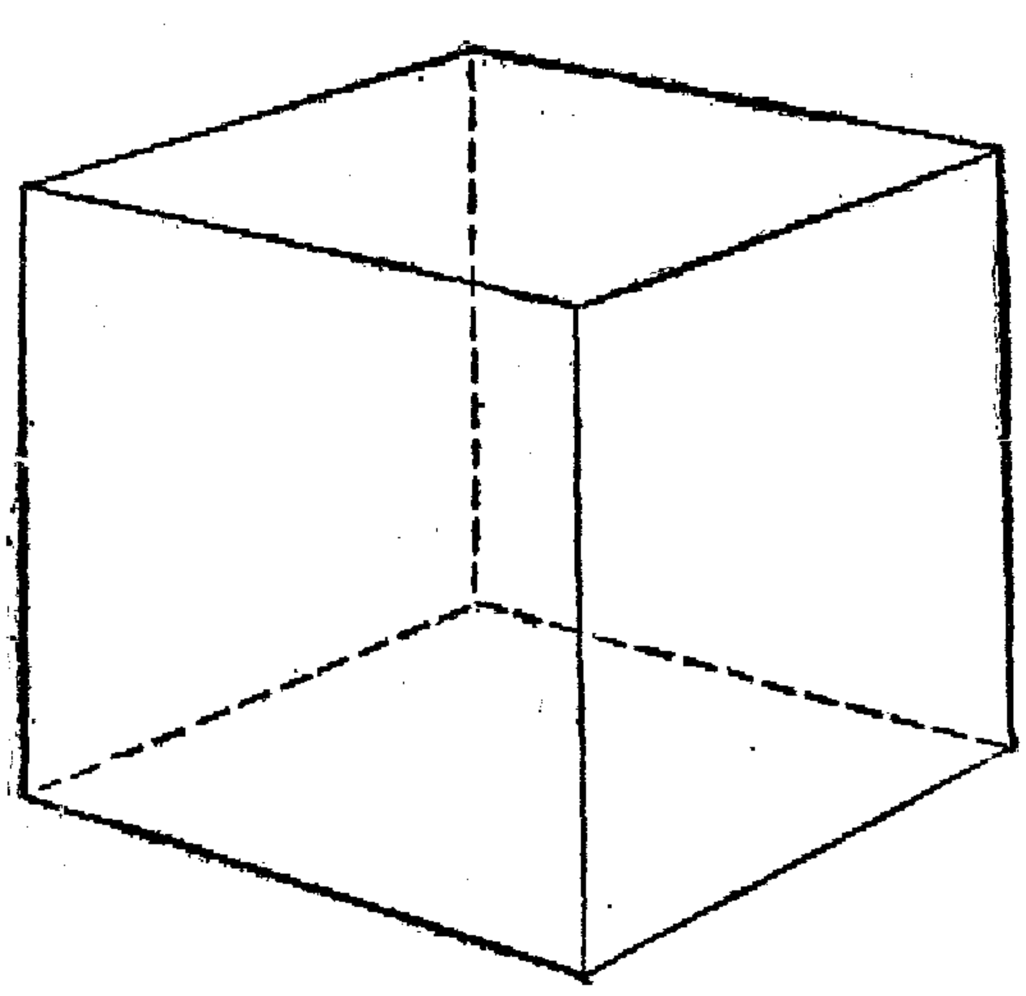

FIG. 4

CUBIC MEASURE

13. Definitions.—A **solid** is a figure that has length, breadth, and thickness. The boundaries of a solid are surfaces. A solid bounded by six squares, as Fig. 4, is a **cube.** If the solid is bounded by six rectangular surfaces it is a **rectangular solid.**

The space included between the bounding surfaces of a solid is called the **cubical contents, capacity,** or **volume** of the solid.

The **unit of volume** is a cube, the edges of which are of a length equal to that of the corresponding unit of length. Thus, 1 **cubic inch** measures 1 inch on each edge; 1 **cubic foot** measures 1 foot on each edge, etc.

TABLE OF CUBIC MEASURE

		Abbreviations
1,728 cubic inches (cu. in.)	=1 cubic foot	cu. ft.
27 cubic feet	=1 cubic yard	cu. yd.
128 cubic feet	=1 cord	cd.
16½ to 25 cubic feet	=1 perch	P.

14. The **standard cord** is a measure of wood; it is a pile 8 feet long, 4 feet wide, and 4 feet high. In some localities it is customary to consider the cord as a pile of wood 8 feet long and 4 feet high, the length of the wood, that is, the width of the pile, being left out of consideration.

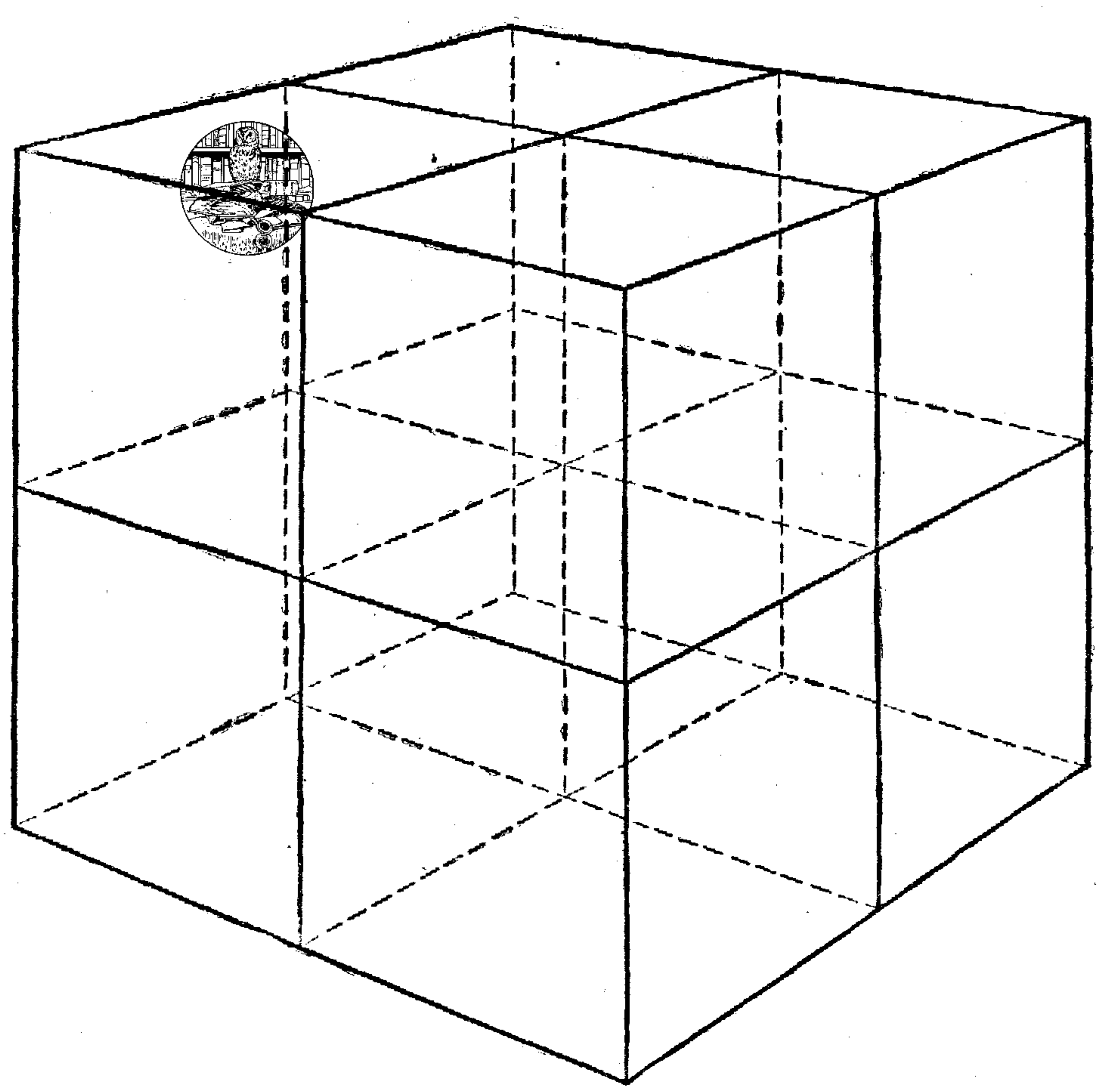

Fig. 5

The **perch,** a measure of stone and masonry, is but rarely used at present. Its cubical contents varies, according to locality, from 16½ to 25 cubic feet. The dimensions will vary accordingly; if the contents is 24¾ cubic feet, the perch would be 16½ feet, or 1 rod, long, 1½ feet wide, and 1 foot high.

15. Rule for Calculating Volume.—When writing the three dimensions of a solid, as 3 feet wide by 2 feet deep by 6 feet long, the word "by" is frequently represented by the symbol $\times$, and the preceding expression is written: 3 ft. $\times$2 ft.$\times$6 ft.

The volume of any rectangular solid is the product of its three dimensions. For example, a box with inside dimensions 2 ft.$\times$4 ft.$\times$8 ft. contains $2\times4\times8=64$ cubic feet; an iron bar 10 feet (or 10×12 inches) long, and with end surfaces that are each 1 inch square contains $10\times12\times1\times1=120$ cubic inches.

A cube 2 feet on each edge is a 2-foot cube, and contains $2\times2\times2=8$ cubic feet, as shown in Fig. 5. A cube 12 inches, or 1 foot, on each edge is 1 cubic foot, or a 12-inch cube, and contains $12\times12\times12=1{,}728$ cubic inches. A cube 3 feet, or 1 yard, on each edge is 1 cubic yard, or a 3-foot cube, and contains $3\times3\times3=27$ cubic feet. Tables of cubic measure can be calculated from tables of linear measure in this manner.

The preceding method of calculating the volume of a rectangular solid may be stated as a rule, as follows:

Rule.—*To find the volume of a cube or a rectangular solid, multiply together the length, breadth, and depth, all expressed in the same units.*

If the dimensions are stated in inches, the volume is given in cubic inches; if in feet, the volume is given in cubic feet, and so on.

EXAMPLE 1.—A sand bin is 14 feet long, 8 feet wide, and 6 feet high. How many cubic feet does it contain?

SOLUTION.—According to the rule, it contains

$$14\times8\times6=672 \text{ cu. ft.} \quad \text{Ans.}$$

EXAMPLE 2.—How many cubic inches of metal are there in a block $12\frac{1}{2}$ inches long, $8\frac{1}{2}$ inches wide, and 4 inches thick?

SOLUTION.—Applying the rule, the volume is

$$12\tfrac{1}{2}\times8\tfrac{1}{2}\times4=\frac{25}{2}\times\frac{17}{2}\times\frac{4}{1}=425 \text{ cu. in.} \quad \text{Ans.}$$

EXAMPLE 3.—A box contains 86,400 cubic inches. What is the contents expressed in cubic feet?

SOLUTION.—According to the table, 1 cu. ft. contains 1,728 cu. in. The problem is, therefore, to ascertain how many times 1,728 is contained in 86,400, or,

$$86{,}400\div1{,}728=50 \text{ cu. ft.} \quad \text{Ans.}$$

16. Board Measure.—A special branch of cubic measure is known as **board measure.** In measuring sawed lumber, the unit of measure is the **board foot,** which is equal to the contents of a board 1 foot square and 1 inch thick. A board foot is, therefore, equal to one-twelfth of a cubic foot. Boards less than 1 inch thick are usually reckoned as though the thickness were 1 inch; but in buying and selling, the actual thickness is considered in fixing the price.

The following rule may be used for finding the number of feet, board measure (B. M.), in a piece of lumber:

Rule.—*To find the number of feet, board measure, in a piece of lumber, multiply together the length, in feet, the width, in inches, and the thickness, in inches, and divide the product by 12, thicknesses less than 1 inch being considered as 1 inch.*

EXAMPLE 1.—How many feet, B. M., are there in a board 14 feet long, 8 inches wide, and 1 inch thick?

SOLUTION.—According to the rule the number of feet, B. M., is equal to $\frac{14\times8\times1}{12}=9\frac{1}{3}$ ft. B. M. Ans.

EXAMPLE 2.—How many feet, B. M., are there in a plank 10 feet 6 inches long, 15 inches wide, and 3 inches thick?

SOLUTION.—Length=10 ft. 6 in.=$10\frac{1}{2}$ ft.=10.5 ft. The number of feet, B. M., is

$$\frac{10.5\times15\times3}{12}=39.375=39\tfrac{3}{8}\text{ ft. B. M.}$$

EXAMPLE 3.—Find the number of feet, B. M., in 20 pieces of siding $\frac{1}{2}$ inch thick, $5\frac{1}{2}$ inches wide, and 10 feet 9 inches long.

SOLUTION.—Since the thickness is less than 1 in., the boards are considered to be 1 in. thick when finding the amount of material in them. Length=$10\frac{9}{12}$ ft.=10.75 ft., width=$5\frac{1}{2}$ in.=5.5 in. The required length is

$$\frac{20\times10.75\times5.5\times1}{12}=\frac{1{,}182.5}{12}=98.5\text{ ft. B. M. Ans.}$$

EXAMPLE 4.—How many feet, B. M., of 2-inch planking are needed for a barn floor 18 ft.×25 ft.?

SOLUTION.—The planks are considered as 25 ft. long. Reducing the width to inches and considering the whole floor as one plank, its width is equal to $18 \times 12 = 216$ in. Proceeding as in the rule, the number of feet required is

$$\frac{25 \times 216 \times 2}{12} = 900 \text{ ft. B. M.} \quad \text{Ans.}$$

EXAMPLE 5.—A board is $9\frac{1}{2}$ ft.$\times$4 in.$\times 1\frac{1}{4}$ in. How many feet, B. M., does it contain?

SOLUTION.—Applying the rule,

$$\frac{9\frac{1}{2} \times 4 \times 1\frac{1}{4}}{12} = 3.96 \text{ ft. B. M., nearly.} \quad \text{Ans.}$$

MEASURES OF CAPACITY

LIQUID MEASURES AND DRY MEASURES

17. Liquid Measure.—The term **capacity** means cubical contents and is used here in connection with measures to indicate the relative amount of space they possess for measuring purposes.

Liquid measure is used for measuring liquids. Liquids in small quantities are measured in pints and quarts; in larger quantities, the gallon is the more common unit, though quantities are sometimes stated in barrels, but rarely in hogsheads. The standard barrel, as used in measuring capacity, contains $31\frac{1}{2}$ gallons, but the barrels ordinarily used vary greatly in size.

One United States standard liquid gallon, known as the *Winchester* or wine gallon, contains 231 cubic inches. One gallon of pure water weighs 8.355, or approximately $8\frac{1}{3}$ pounds. One cubic foot contains 7.481, or approximately 7.5, gallons, and 1 cubic foot of water, at 62° Fahrenheit, weighs $62\frac{1}{2}$ pounds, nearly.

In some English-speaking countries the beer or ale gallon of 282 cubic inches' capacity is used for measuring the liquids mentioned.

The British imperial gallon, used in Canada, contains 277.463 cubic inches, and the weight of such a gallon of pure water is 10 pounds. One imperial gallon equals approximately 1.2 United States liquid gallons.

TABLE OF LIQUID MEASURE

English System

		Abbreviations
4 gills (gi.)	=1 pint	pt.
2 pints	=1 quart	qt.
4 quarts	=1 gallon	gal.
$31\frac{1}{2}$ gallons	=1 barrel	bbl.
2 barrels	=1 hogshead	hhd.

18. Dry Measure.—Dry articles, such as fruit, grain, vegetables, etc. are measured by **dry measure.** The standard unit is the United States bushel, otherwise known as the *Winchester bushel,* which contains 2,150.42 cubic inches; and $\frac{1}{2}$ peck (dry gallon) contains 268.8 cubic inches.

One British imperial bushel contains 2,219.704 cubic inches, or 1.0322 United States bushels.

The quart used in dry measure is not the same as the quart used in liquid measure. The dry quart contains 67.2 cubic inches, and the liquid quart only $57\frac{3}{4}$ cubic inches.

TABLE OF DRY MEASURE

		Abbreviations
2 pints (pt.)	=1 quart	qt.
8 quarts	=1 peck	pk.
4 pecks	=1 bushel	bu.

19. The application of the preceding tables is shown in the following examples:

Example 1.—How many quarts are there in 2 barrels?

Solution.—According to the table of liquid measure there are 4 qt. in 1 gal., and $31\frac{1}{2}$ gal. in 1 bbl. Therefore, in 2 bbl. there are

$$2 \times 31.5 = 63.0 \text{ gal.}$$

and

$$63 \times 4 = 252 \text{ qt. Ans.}$$

Example 2.—How many hogsheads will be required for 315 gallons of molasses?

Solution.—According to the table of liquid measure, 1 hhd.=2 bbl., and 1 bbl.=$31\frac{1}{2}$ gal. In 2 bbl. there are $2 \times 31\frac{1}{2} = 63$ gal. Therefore, 315 gal. require

$$315 \div 63 = 5 \text{ hhd. Ans.}$$

Example 3.—A man bought 10 bushels of potatoes and intends to sell them by the quart. How many quarts of potatoes has he on hand?

SOLUTION.—According to the table of dry measure, 1 bu.=4 pk., and 1 pk.=8 qt., so that in 1 bu. there are 4×8=32 qt.; in 10 bu. there are

$$10 \times 32 = 320 \text{ qt.} \quad \text{Ans.}$$

MEASURES OF WEIGHT

AVOIRDUPOIS AND TROY MEASURES

20. Standard Units of Weight.—There are two English measures of weight in general use, the **avoirdupois** (av.) **weight,** for coarse, heavy substances, such as coal, iron, copper, hay, and grain, and the **Troy weight,** for finer and more valuable substances, such as gold, silver, and jewels. Besides there is the **apothecaries' weight,** which is used by physicians in prescribing and by druggists in compounding medicines. Medicines are bought and sold by avoirdupois weight.

TABLE OF AVOIRDUPOIS WEIGHT

		ABBREVIATIONS
27.3438 grains (gr.)	=1 dram	dr.
16 drams	=1 ounce	oz.
16 ounces (oz.)	=1 pound	lb.
100 pounds	=1 hundredweight	cwt.
20 hundredweight / 2,000 pounds	=1 ton	T.

TABLE OF TROY WEIGHT

		ABBREVIATIONS
24 grains (gr.)	=1 pennyweight	dwt.
20 pennyweights	=1 ounce	oz.
12 ounces	=1 pound	lb.

TABLE OF APOTHECARIES' WEIGHT

		ABBREVIATIONS
20 grains (gr.)	=1 scruple	sc.
3 scruples	=1 dram	dr.
8 drams	=1 ounce	oz.
12 ounces	=1 pound	lb.

21. One **avoirdupois pound** equals approximately 1.2153 **Troy pounds,** the former containing 7,000 Troy grains and the latter 5,760 grains. One avoirdupois ounce