ALGEBRA

(PART 1)

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GENERAL NUMBER

NOTATION AND NUMERATION

1. Algebra as well as arithmetic deals with number. The laws of algebra are based on the laws of arithmetic. But, in passing from ordinary arithmetic to algebra, the mode of representing number and the manner of dealing with it are extended.

2. Symbols of General Value.—In algebra, letters as well as figures are used as symbols to represent numbers. When so used, letters are symbols of number in *general*, that is, a letter may represent *any number whatever*. Throughout any problem, however, the same letter must represent the same number.

In arithmetic we say a man earns 30 dollars a month; in algebra we say a man earns a dollars, or x dollars, a month.

3. Numbers represented by letters are reasoned about and operated upon in the same manner as numbers represented by figures.

SYMBOLS OF OPERATION AND THEIR USES

4. Sign of Addition.—The sign +, read *plus*, when placed between two numbers, indicates that the numbers are to be added. Thus, $2+5$ indicates that 2 and 5 are to be added. Similarly, $a+b$ indicates that the numbers for which a and b stand are to be united in one sum, or *thought of* as added.

5. Sign of Subtraction.—The sign −, read *minus*, when placed between two numbers, indicates that the number on the right is to be subtracted from the number on the left. Thus, $7-4$ indicates that 4 is to be subtracted from 7. Similarly, $a-b$ indicates that b is to be subtracted from a.

6. When numbers are represented by figures, indicated operations can nearly always be *actually performed;* but when numbers are denoted by letters, operations can usually be only *indicated.*

Thus, if John owes James \$10 and Henry \$22, his total debt is \$10 + \$22, or \$32. But if John owes James a dollars and Henry b dollars, his debt to both can only be indicated, as $a+b$ dollars.

7. Sign of Multiplication.—The sign of multiplication is × and is read *multiplied by* or *times.* Thus, 6×5 is read *six multiplied by five* or *six times five.*

8. The product of two or more letters is indicated by writing the letters together with no intervening sign. Thus, ab means $a\times b$. In all cases where no ambiguity can arise, the multiplication sign is omitted. Thus, $2\times a$ is written $2a$.

9. When a given number is the product of two or more numbers, each of these is a **factor** of the given number.

10. In an indicated product, any part of the product is the **coefficient** of the rest of the product.

Thus, in $2ab$,

2 is the coefficient of ab;
$2a$ is the coefficient of b;
$2b$ is the coefficient of a.

A coefficient expressed in figures is a **numerical coefficient,** and a coefficient expressed in letters is a **literal coefficient.** Thus, in $3x$, 3 is the numerical coefficient of x, and in ax, a is the literal coefficient of x.

When no numerical coefficient is written, 1 is understood. Thus, a means $1a$ and cd means $1cd$.

11. Sign of Division.—The sign $\div$, read *divided by*, when placed between two numbers indicates that the first number is to be divided by the second. Thus, $a \div b$ indicates that a is to be divided by b.

Division in algebra is generally indicated by writing the dividend over the divisor with a horizontal line between them. Thus, $\frac{a}{b}$ is read *a over b*, and denotes that a is to be divided by b.

ILLUSTRATIVE EXAMPLES

12. The use of letters to represent number in general is shown in the following examples:

EXAMPLE 1.—If a boy has 5 marbles and buys 2 more, he has $5+2$ marbles. If he has a marbles and buys 7 more, he has $a+7$ marbles. If he has x marbles and buys y more, he has $x+y$ marbles.

EXAMPLE 2.—If a man has 10 dollars and spends 3 dollars, he has left $10-3$ dollars. If he has m dollars and spends 5 dollars, he has left $m-5$ dollars. If he has r dollars and spends s dollars, he has $r-s$ dollars left.

EXAMPLE 3.—If a man buys 7 horses at 150 dollars each, he pays 7×150 dollars for the horses. If he buys m horses at 150 dollars each, he pays $150\,m$ dollars for the horses. If he buys x horses at y dollars each, he pays xy dollars for the horses.

EXAMPLE 4.—If a man sells 4 barrels of apples for 12 dollars, he receives $\frac{12}{4}$ dollars per barrel. If he sells b barrels for 16 dollars, he receives $\frac{16}{b}$ dollars per barrel. If he sells n barrels for m dollars, he receives $\frac{m}{n}$ dollars per barrel.

EXAMPLE 5.—If a man walks 20 miles a day for 2 days and then rides 40 miles a day for 3 days, he travels in $2+3$, or 5, days $(2\times 20)+(3\times 40)$ miles. If he walks x miles a day for y days and then rides a miles a day for b days, he travels in the $y+b$ days $xy+ab$ miles.

EXAMPLE 6.—If a man buys 5 yards of cloth at 60 cents a yard and sells it at 80 cents a yard, he gains $(5\times 80)-(5\times 60)$ cents. If he buys x yards of cloth at y cents a yard and sells it at a gain for z cents a yard, he gains $xz-xy$ cents.

EXAMPLE 7.—If in a number of two digits, the digit in the tens place is 4 and the digit in the units place is 5, the number is $(10\times 4)+5$. If the digit in the tens place is a and the digit in the units place is b, the number is $10a+b$.

13. The preceding examples show that the reasoning is the same whether the numbers are represented by figures or by letters, either wholly or partly.

EXAMPLES FOR PRACTICE

1. A box is a feet wide and its length is b feet more than its width; how long is it? Ans. $a+b$ feet

2. A man had m acres of land and sold n acres; how many acres had he left? Ans. $m-n$ acres

3. A boatman rowed a miles up a river, was towed c miles farther, and then floated 12 miles down stream; how far was he from the place of starting? Ans. $a+c-12$ miles

4. A number x is divided into two unequal parts; if the less is y, what is the greater? Ans. $x-y$

5. A merchant bought c bushels of potatoes at d dollars a bushel and sold them at b dollars a bushel. (*a*) If b is greater than d, did he gain or lose? (*b*) How much? Ans. { (*a*) He gained; (*b*) $bc-cd$ dollars }

6. A boy worked a hours a day for x days at y cents an hour. With a part of the money he bought b marbles at c cents each. (*a*) How much money did he earn? (*b*) How much money was left after buying the marbles? Ans. { (*a*) axy cents; (*b*) $axy-bc$ cents }

7. A train travels x miles in y hours, what is the rate in miles per hour? Ans. $\frac{x}{y}$

8. A man paid a dollars for corn at b dollars a bushel; how many bushels did he buy? Ans. $\frac{a}{b}$

9. A man drives 3 hours at the rate of 10 miles an hour, how long will it take him to walk back at the rate of 6 miles an hour?

Ans. $\frac{10 \times 3}{6}$, or 5 hours

10. A man rides x hours at the rate of y miles an hour; how long will it take him to walk back at the rate of z miles an hour?

Ans. $\frac{xy}{z}$ hours

14. Symbols of Aggregation.—The signs of aggregation are the **parenthesis** ( ), the **bracket** [ ], the **brace** { }, the **vinculum** ——, and the **bar** |. Each indicates that the several numbers to which it is affixed are to be regarded as a single number. Thus, $(x+y)$, $[x+y]$, $\{x+y\}$, $\overline{x+y}$, and $\left.\begin{matrix}x\\+y\end{matrix}\right|$, each indicates that $x+y$ is to be treated as one number.

Whenever it is possible, the operations indicated by the signs within a sign of aggregation should be performed first. Thus, $(2+3)\times 4$ means that the sum of 2 and 3 is to be multiplied by 4; hence, $(2+3)\times 4 = 5\times 4 = 20$. In like manner, $(7-3)\times 2$ means $4\times 2 = 8$.

15. The sign of multiplication is usually omitted between two factors one of which is enclosed by a sign of aggregation.

Thus, $3\times(2+5)$ is written $3(2+5)$;
$(a+b)\times c$ is written $(a+b)c$, or $c(a+b)$;
$(a+b)\times(c+d)$ is written $(a+b)(c+d)$.

16. Exponents.—In such expressions as 3^2 and a^2b^3, the small figures ^{2} and ^{3} written at the right and a little above, are **exponents.** An exponent shows how many times the number to which it is attached is to be taken as a factor.

Thus, 3^2 means 3×3;
2^3 means $2\times 2\times 2$;

and so on.

It is important to notice that 2^1 is the same as 2. In like manner, a^1 is the same as a, a^2 means aa, a^3 means aaa, and so on.

a^1, or a, is the *first power* of a;
a^2, or aa, is the *second power* of a;
a^3, or aaa, is the *third power* of a;

and so on.

a^2 is read *a square.*
a^3 is read *a cube.*
a^4 is read *a fourth power*, or *a fourth.*
a^5 is read *a fifth power*, or *a fifth.*
a^n is read *a nth power*, or *a nth.*

17. When a number enclosed in parenthesis is to be taken in a product more than once as a factor, an exponent is used to indicate how many times it is to be taken. Thus, $(x+y)^2$ means $(x+y)(x+y)$, and is read *the square of x plus y.* Also, $2ab^2(a-b)^3$ means $2ab^2(a-b)(a-b)(a-b)$ and is read *two ab square times the cube of a minus b.*

18. A **mathematical expression,** or simply an **expression,** is any combination of symbols that represents a number. Thus, 13, a, $5b$, $a+b$, a^2+b^2, $2a+3b-7c^2$ are expressions because each represents a number.

19. Terms.—In such an expression as $a^2-2ab+b^2$ or $ax^2+(b+c)x+d$, the parts of which it is composed together with the sign that precedes each are **terms.**

Thus, the first of the foregoing expressions contains the three terms, $+a^2$, $-2ab$, and $+b^2$, and the second contains the three terms, $+ax^2$, $+(b+c)x$, and $+d$.

20. The sign $+$ or $-$ that precedes a term is the **sign of the term.**

21. A **positive term** is a term that has the sign $+$ before it, either expressed or understood.

A **negative term** is a term that has the sign $-$ before it.

The sign $+$ is usually omitted before a monomial and the first term of a polynomial. Thus, the expression

$2a^2 - 3ab + 5b^2$ consists of the positive terms $2a^2$ (or $+2a^2$) and $+5b^2$, and the negative term $-3ab$.

22. **Like** or **similar terms** are terms that have the same letters affected by the same exponents. Thus, $2ab^2$, $3ab^2$, and $-7ab^2$ are like terms.

23. **Unlike** or **dissimilar terms** are terms that differ in their literal parts. Thus, $5ab$ and $5ab^2$ are unlike terms.

MONOMIALS AND POLYNOMIALS

24. A **monomial** is an algebraic expression consisting of but one term. Thus, $6a$ and $5a^2b^3c$ are monomials.

A **polynomial** is an algebraic expression consisting of two or more terms. Thus, $x+y$ and $a-b+x^2y-8z$ are polynomials.

A **binomial** is a polynomial of two terms. Thus, $x+y$ is a binomial.

A **trinomial** is a polynomial of three terms. Thus, $ab+xy-z$ is a trinomial.

25. The polynomial $a + \frac{a^2b}{mn^2} + 2a^3 - 3a^4b^5$ is read *a, plus a square b over mn square, plus two a cube, minus three a fourth b fifth.*

Considerable care is required in reading expressions containing symbols of aggregation. For example, the monomial $4(a-b)$ must not be read *four a minus b*, for this would be understood to mean $4a-b$; but $4(a-b)$ should be read *four times a minus b*, when it will be understood that 4 multiplies the whole quantity $a-b$, since *times* or *into* is not used in reading such a product as $4a$.

In reading polynomials containing symbols of aggregation, a distinct pause should be made after each term. Thus, the polynomial $5x^3 + (a+b)x^2y - (a^2-ab+b^2)xy^2 + (a^3+b^3)y^3$ should be read *five x cube, plus the quantity a plus b times x square y, minus the quantity a square minus ab plus b square* *times xy square, plus the quantity a cube plus b cube times y cube.* The expression $5(a+b)(x+y)-ax$ should be read *five times the quantity a plus b times the quantity x plus y, minus ax.*

26. Sign of Equality.—The sign of equality $=$, read *is equal to*, is used in algebra, as in arithmetic, to show that the expression before it is equal to the expression after it. Thus, $a+b=10$ denotes that the sum of the two numbers a and b is equal to 10.

EXAMPLES FOR PRACTICE

Read the following expressions:

1. $6ab$	6. x^2-2ax	11. $ax+cy+mn$
2. $4x^2y$	7. $3(x-2y)$	12. $abc+(a-b)y+xy$
3. $7m^2n$	8. $x^2(a-b)^2$	13. $(ax+bm)y-8$
4. $10xyz^3$	9. $a^2(2+6b)$	14. $3x+2y-(a+b)z$
5. $\frac{a^2b}{xy}$	10. $\frac{a+2x}{a-2x}$	15. $\frac{(m+n)^2}{m-n}$

Write the following as algebraic expressions:

16. Five a square times the square of x plus y.

17. The quantity five a minus b times the square of x minus three y. Ans. $(5a-b)(x-3y)^2$

18. x square plus two times the quantity a minus b times xy square plus the cube of a plus b.

19. Read the equation $x^2+(2a+b)x+c=4x+10$.

EVALUATION

27. The result obtained by substituting particular values for the letters that occur in an expression and performing the indicated operations, is the **numerical value** of the expression. The process of finding the numerical value of an expression is called **evaluating** the expression.

EXAMPLE 1.—When $a=4$ and $b=3$, find the numerical value of the expression $ab+2b$.

SOLUTION.—Substituting 4 for a and 3 for b, gives

$$ab+2b=4\times3+2\times3$$
$$=12+6=18. \quad \text{Ans}$$